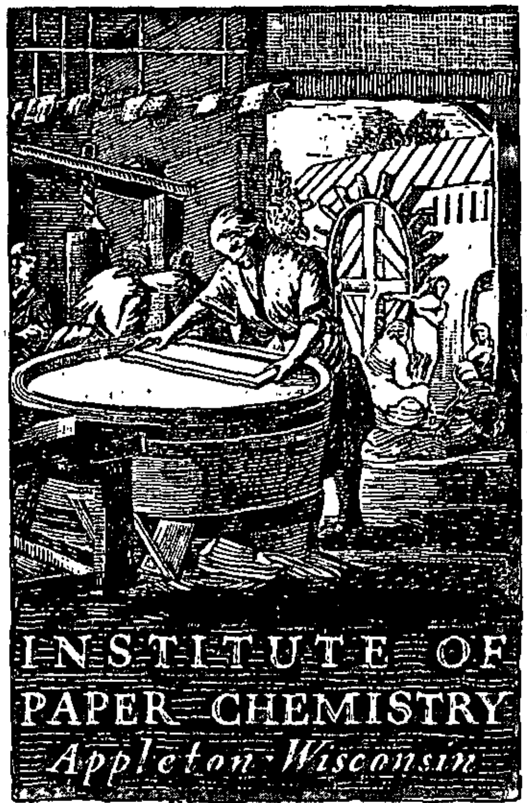

VARIABILITY OF 42-LB. KRAFT LINER

Project 1108-21

Progress Report One

to

FOURDRINIER KRAFT BOARD INSTITUTE, INC.

November 7, 1958

THE INSTITUTE OF PAPER CHEMISTRY

Appleton, Wisconsin

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TABLE OF CONTENTS

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SUMMARY

At the request of the Fourdrinier Kraft Board Institute, Inc., a statistical investigation was performed using 42-lb. linerboard samples submitted in connection with the linerboard baseline study. The properties of interest were basis weight, caliper, bursting strength, and Elmendorf tear. It was desired that the variability of a given property be approached from two standpoints--that is, (1) the variability within samples (reels in this case) and, (2) the variability between samples (reels). It should be noted that because of the manner in which liner samples are obtained, the within-reel variability might with equal justification be termed across-machine variability.

The analysis reported herein was performed using the baseline study data gathered during 1957. A total of 1076 samples distributed among 17 mills were examined for each of the above properties.

For purposes of comparison, the within reel variability was expressed as per cent two standard deviations in this report. This has the significance that if production were maintained at a given quality level, approximately 95% of the individual test values will be within ± 2 per cent standard deviation of the grand average. It may be recognized, however, that any deviations from "ideal" statistical control--such as accidental or planned changes in production level or uniformity--tend to produce the result that a <u>lesser</u> percentage of the values fall within the stated limits. Control charts and frequency distributions were prepared for each mill and for each test property in order to graphically illustrate the significance

of the above and other interesting aspects of the data. Examination of the control charts indicated that "ideal" statistical control was not usually achievod; however, in some instances a relatively close approach could be observed.

The above facts indicated that many "populations" of reel average were present within the production from even one mill. Frequency distributions of the reel averages frequently indicated the presence of bi-modal or tri-modal distributions rather than the bell-shaped distribution characteristics of the normal frequency curve. For reasons such as these, the estimates of between reel variability were invariably greater than the estimates of within reel variability. However, because the between reel variability represents the dispersion of a number of differing populations of reel averages, the use of a "single figure" for characterizing between reel variability based on the concept of a single homogeneous population is not rigorously justified. From a practical standpoint it may serve some useful purposes as long as its limitations are understood.

INTRODUCTION

Early this year, The Institute of Paper Chemistry was requested to outline a study for the purpose of determining the variability associated with the 42-lb. linerboard production as represented by samples submitted in connection with the linerboard baseline study. The properties of interest were basis weight, caliper, bursting strength and tearing strength. In accordance with the above, it was proposed that the baseline study data for January, 1957 through December, 1957 be analyzed to determine:

1. The variability of individual test results about the reel averages for each mill.

2. The variability of reel averages about the over-all reel average for each mill.

In contrast to previous studies along these lines it was requested that the statistical work be based on the standard deviation of the individual sample averages rather than on the range.

Also it was requested that frequency distributions of reel averages be compiled for all properties of interest.

PROCEDURE

The baseline study data during the period January-December, 1957 included 1076 samples of kraft linerboard submitted by seventeen mills. It should be emphasized that the samples submitted represent reel samples rather than roll samples.

1. AVERAGE VARIABILITY IN REELS

The variability within a reel is an indication of the dispersion of the individual test values about the reel average for a given property. The variability within a reel may be estimated in terms of the standard deviation. The standard deviation for a given sample may be computed by means of Equation (1):

$$s = \sqrt{\frac{N\Sigma X^2 - (\Sigma X)^2}{N^2}} \qquad (1)$$

where s = sample standard deviation

N = number of individual test determination in average

X = individual readings

As mentioned, s is merely an estimate of the deviation or spread of the individual values in a sample about the sample average. For example, in the case of reel bursting strength, s is an estimate of the spread or dispersion of the individual bursting strength readings about the reel average bursting strength. In order to estimate the average variability to be expected within a reel for a given mill, the sample standard deviations s for each mill were averaged to obtain an average sample standard deviation $\bar{s}$. The average sample standard deviation $\bar{s}$ was then used to estimate the population standard deviation, σ, for each mill by Equation 2.

$$\sigma = \frac{\bar{s}}{c_2} \qquad (2)$$

where c_2 = ratio of average standard deviation for sample of given size to the standard deviation of the population from which the sample was taken.

In the present study, sigma, σ, is taken as an estimate of the average "within reel" variability. This has the significance that if production for a given mill were maintained at a given quality level and hence in statistical control that it would be anticipated that approximately 95% of the individual reel values would fall within a $\pm 2\sigma$ of the composite reel average and hence the same approximation as regards reel averages. Since sigma is an estimate of the variability about the population average for a given mill's samples over a given production period and each mill is operating at a slightly different average level, the variability is expressed as a per cent average variability by dividing each mill's sigma by its corresponding population average and then multiplying by 100. To illustrate the manner in which the quality level for each mill fluctuated during 1957 on the basis of the data available, quality control charts were prepared using 2 standard error limits. By definition, standard error is equal to sigma divided by the square root of N. The number of reel averages falling outside these limits is an indication of how the quality average fluctuated throughout the period.

2. VARIABILITY BETWEEN REEL AVERAGES

The "between reel" averages variability was obtained by computing the standard deviation of the sample averages about the composite or population averages for each mill. This was accomplished by substituting the reel averages for a given mill in Equation (1) to calculate the standard deviation of the averages (standard error). Again for comparison purposes, this variability is expressed in per cent at the 2 standard error level.

Frequency distribution charts were compiled for each of the properties desired to illustrate the distribution of the sample averages for each test property for each mill.

DISCUSSION OF RESULTS

GENERAL

In using the term "reel," it may be recalled that liner baseline study samples are obtained as strips across the full width of the machine. From this standpoint, each sample average and the dispersion of the individual test values about that average will reflect the differences in test properties which occur across the machine width. Because of the manner of sampling, therefore, "within reel variability" might, with equal justification, be termed across-machine variability.

A reel sample can, at best, be representative of production quality at only one instance in time. Differences between reel averages will reflect the random or planned changes in quality level which occur over the period of time between reels. Between reel variability, therefore, will be a function to some extent of the within reel variability but, more importantly, it will be a function of any changes in production quality over the time period.

To illustrate the above remarks, Figure 1 represents a hypothetical process in which important shifts in reel averages occur relative to the within reel variability. In preparing the figure, the assumption has been made that the within reel variability was essentially in control over the entire period of time--that is, only shifts in reel average were considered to be present.

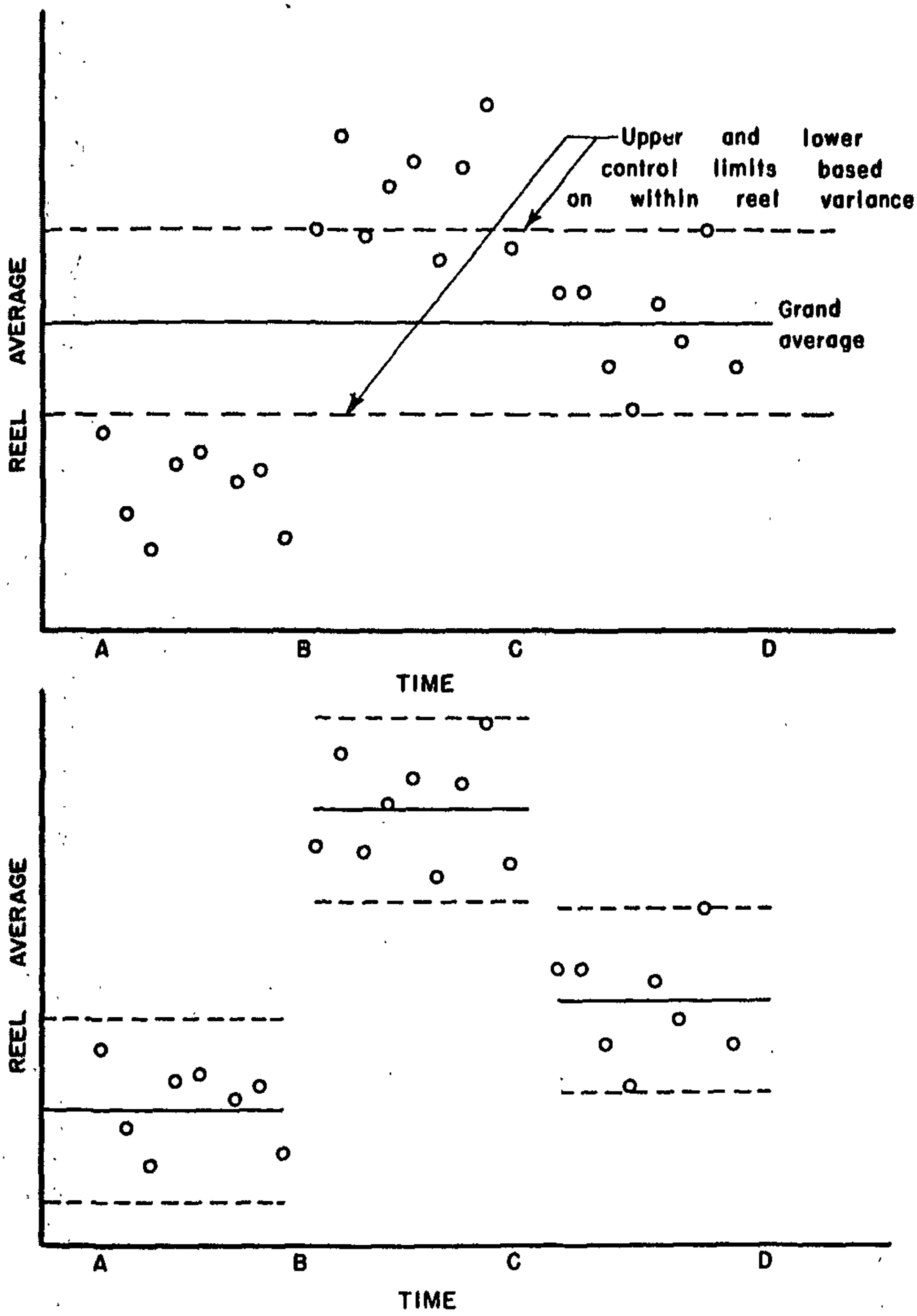

Figure 1. Representation of a Process in Which Significant Shifts in Reel Averages Occur Over a Period of Time.

Referring to the upper graph in Figure 1, during the time period AB reel averages ran at a relatively low level. This was followed in time period BC by a series of relatively high reel averages and finally in period CD by a number of reel averages near the grand average. The grand average of all the points is shown together with upper and lower control limits based on the within reel variability. It may be inferred from the upper graph that three separate reel populations were present. For example, as shown in the lower graph, the reel averages for the time period AB could be considered to constitute a homogeneous population with its own average and central limits. As shown in the lower graph, the limits for each population are an identical distance from each group averages. This followed as a consequence of the assumption that the within reel variance was constant over the entire period of time. In general, however, shifts in process variance could also occur and the limits for each population would be different.

It is apparent from the figure that the grand average has little significance. By the same token, if the process variance changed from time to time, an average for the variance would also have little significance. It may be remarked here, however, that the plot of the data in control chart form for each mill seems to indicate that the cross-machine or within-reel variance is usually more constant than the reel averages. In many instances, the within reel variance appears to be essentially in statistical control.

The between reel variability as calculated herein consisted of determining the standard deviation of the reel averages about the over-all or grand average. From one standpoint, the tacit assumption is made that the reel averages form a homogeneous population distributed in conformance with the normal distribution curve. The value of the resulting figure for the standard deviation of the average loses much of its significance where several or many populations are present. For example, the distribution of averages in Figure 1 would approach a tri-modal (tri-peaked) distribution curve. Many of the frequency distributions of reel averages shown in later pages depart from the "ideal" to a marked extent. For this reason, extra care should be exercised in the interpretation of probabilities from the between-reel variances summarized herein.

Certain other aspects of the data deserve consideration. For example, it may be somewhat dangerous to characterize production quality on the basis of a relatively small number of samples per mill per month. The difficulty is compounded when the sampling is erratic, that is, many samples one month and one or relatively few in another month. A second and more subtle question arises regarding the method of sampling. It is conceivable, e.g., that certain mills may forward samples from reels of selected quality. Such a practice might well give an appearance of superior control. The seriousness of these problems depends to some extent on the use to which the data are put. In general, it is believed that certain interesting features of mill quality control are illustrated by the data and that certain general conclusions of value to the industry may be derived from the study.

BASIS WEIGHT

The comparisons of within and between-reel variability are summarized in Table I and the frequency distribution of the reel averages are shown in Table II for each mill. In Table I it may be noted that since Mill Q

TABLE I

COMPARISON OF THE VARIABILITY WITHIN AND BETWEEN REELS OF BASIS WEIGHT BY MILLS

Mill	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	Composite
Within Reel Variability																		
No. of samples	62	38	64	98	41	29	96	38	98	56	68	92	108	56	74	52	6	1076
Grand av., $\bar{\bar{X}}$	42.6	42.8	43.1	43.5	42.2	43.3	42.7	42.9	43.6	42.8	43.0	43.8	43.7	42.8	42.7	43.3	42.4	43.0
Av. standard deviation, $\bar{\sigma}$	0.620	0.678	0.380	0.422	0.446	0.838	0.544	0.482	0.856	0.452	0.682	0.658	0.474	0.686	0.488	0.572	0.300	0.573
Estimated population, standard deviation, σ	0.662	0.726	0.406	0.450	0.476	0.896	0.582	0.516	0.914	0.482	0.728	0.704	0.506	0.732	0.522	0.612	0.328	0.612
Per cent two standard deviation	3.11	3.39	1.88	2.07	2.26	4.14	2.73	2.41	4.19	2.25	3.39	3.21	2.32	3.42	2.44	2.83	1.55	2.85
Two standard error, $2\,\sigma/\sqrt{N}$	0.382	0.420	0.234	0.260	0.274	0.518	0.336	0.298	0.528	0.278	0.420	0.406	0.292	0.422	0.302	0.354	0.190	0.354
Per cent two standard error	0.90	0.98	0.54	0.60	0.65	1.20	0.97	0.69	1.21	0.65	0.98	0.93	0.67	0.99	0.71	0.82	0.45	0.82
Two S.E. limits about $\bar{\bar{X}}$	42.2-43.0	42.4-43.2	42.9-43.3	43.2-43.8	41.9-42.5	42.8-43.8	42.4-43.0	42.6-43.2	43.1-44.1	42.5-43.1	42.6-43.4	43.4-44.2	43.4-44.0	42.4-43.2	42.4-43.0	43.0-43.8	42.2-42.6	42.6-43.4
Two S.E. limits about $\bar{\sigma}$	0.350-0.890	0.382-0.974	0.214-0.546	0.236-0.606	0.252-0.640	0.472-1.204	0.306-0.782	0.272-0.692	0.482-1.230	0.256-0.648	0.384-0.980	0.370-0.946	0.268-0.680	0.388-0.984	0.274-0.702	0.322-0.822	0.166-0.434	0.323-0.823
Between Reel Variability																		
Two standard error, $2\,\sigma_r/\sqrt{N}$	1.664	1.554	1.560	1.014	1.572	1.954	1.378	1.132	1.422	1.266	1.624	1.568	2.048	1.182	1.394	1.498	0.868	1.452
Per cent two standard error	3.91	3.63	3.62	2.33	3.73	4.51	3.23	2.64	3.26	2.96	3.78	3.58	4.69	2.76	3.26	3.46	2.05	3.38

TABLE II

FREQUENCY DISTRIBUTION OF BASIS WEIGHT AVERAGES

Basis Wt., lb./1000 sq.ft.	A	B	C	D	E	F	G	H	I	J	Mills K	L	M	N	O	P	Q	Total	Per Cent	Cumulative Total	Cumulative, %
45.8-45.9													1					1	0.1	1	0.1
45.6-45.7						1			1			1	5					8	0.7	9	0.8
45.4-45.5						1			1		2	2	1					7	0.6	16	1.5
45.2-45.3												1	4					5	0.7	21	2.0
45.0-45.1									2			2	2			1		7	0.6	28	2.6
44.8-44.9				1		1			2			4	6					14	1.3	42	3.9
44.6-44.7									3			5	2					10	0.9	52	4.8
44.4-44.5					1	3			4			3	7			2		20	1.9	72	6.7
44.2-44.3	2	1	1	4			2		7	1		9	11			7		45	4.2	117	10.9
44.0-44.1	3	2	9	15	2	2	4		13		1	12	11	1	2	8		85	7.9	202	18.8
43.8-43.9	1	1	9	12			4	1	6	5	6	<u>10</u>	7	3	1	7		73	6.8	275	25.6
43.6-43.7	1	3	6	19		3	7	4	<u>16</u>	3	6	12	<u>9</u>	2	8			99	9.2	374	34.8
43.4-43.5	5	4	7	<u>16</u>		2	2	6	15	3	5	5	7	7	7			91	8.5	465	43.2
43.2-43.3	3	2	3	11	1	<u>2</u>	6	3	4	9	11	4	5	6	6	<u>5</u>		81	7.5	546	50.7
43.0-43.1	6	3	<u>2</u>	8	1	4	7	4	8	4	<u>8</u>	9	5	5	9	9	1	93	8.6	635	59.4
42.8-42.9	3	<u>5</u>	6	4	3	3	7	<u>9</u>	6	<u>4</u>	5	2	5	7	5	4	1	79	7.3	718	66.7
42.6-42.7	<u>8</u>	3	3	4	2	2	<u>15</u>	3	4	6	4	3	3	<u>4</u>	<u>1</u>	2		67	6.2	785	73.0
42.4-42.5	5	3	2	1	1		10		3	6	3	2	5	8	1	3	<u>1</u>	54	5.0	839	78.0
42.2-42.3	8	1	3	1	<u>4</u>	1	10	2		5	5	5	6	6	11			68	6.3	907	84.3
42.0-42.1	7	2	8	1	9	2	11	3	2	8	5	1	4	2	11	3	3	82	7.6	989	91.9
41.8-41.9	1	4	3	1	7	2	5	1		2	4		1	5	9	1		46	4.3	1035	96.2
41.6-41.7	5	3	2		4		4	2	1		2				3			26	2.4	1061	98.6
41.4-41.5	1				1		2											4	0.4	1065	99.0
41.2-41.3					2													2	0.2	1067	99.2
41.0-41.1	1	1			1													3	0.3	1070	99.4
40.8-40.9					2						1							3	0.3	1073	99.7
40.6-40.7													1					1	0.1	1074	99.8
40.4-40.5																					
40.2-40.3	2																	2	0.2	1076	100.0
Total	62	38	64	98	41	29	96	38	98	56	68	92	108	56	74	52	6	1076			

Note: Underlined values are the grand average.

submitted only six samples during the year, no firm conclusions may be drawn from such a limited number of samples; the values obtained in analysis of their data have been excluded from consideration in the following discussion.

The values of within-reel variability for the remainder of the mills may be compared in terms of the per cent two standard deviation. It may be recalled that if production were maintained at a given quality level throughout the year, approximately 95% of the individual reel values would be expected to fall within ± 2% standard deviation of the population average ($\bar{X}'$) and hence reel average. Referring to Table I it may be noted that Mills I and F exhibited the highest within-reel or cross-machine variability (4.19 and 4.14%, respectively) while Mills C and D exhibited the lowest within reel variabilities (1.88 and 2.07%, respectively).

The estimates of between-reel variability are also expressed in Table I on the per cent two standard error basis. In Table I it may be observed that the between-reel variabilities range from 2.33% for Mill D to a high of 4.69% for Mill M.

Frequency distributions of the individual reel averages are given in Table II for each mill. While the number of samples for each mill is usually not large enough to characterize the distributions, the data do indicate that the reel averages usually spread over a relatively large range rather than clustering closely about the over-all average. In a number of cases such as for Mills C or P, bi-modal or bi-peaked distributions are indicated. Similar tendencies may be noted in a number of the other distributions.

The within-reel standard deviation, as mentioned previously, is a measure of the dispersion of the individual values about the average. It may also be used to estimate the dispersion of averages about a grand average--in other words, for this purpose it is converted to standard error by dividing by the square root of N (the number of readings entering any one average). If a homogeneous population of reel averages were involved, 95% of the reel averages would be expected to fall within ± 2 standard errors of the average. If a homogeneous population is not present, a smaller percentage of the reel averages will fall inside these limits.

As calculated herein, the between reel variability was determined as a measure of the actual dispersion of reel averages about the grand average--that is, in the form of a standard error. Two measures of the dispersion of reel averages may, therefore, be obtained from the data--the first based on the within-reel variability and the second based on the actual dispersion of the averages. If ideal statistical control were present, it would be expected that equivalent results would be obtained by either method. If the differences between reel averages are greater than would be expected in the basis of the within-reel variability, the between-reel variability will be greater than the within-reel variability (both expressed in standard error form).

To illustrate these remarks, the following results were obtained for Mill A in Table I.

	Within	Between	Ratio
Per cent two Standard error	0.90	3.91	4.35
Two standard error limits	±0.382	±1.664	

In this example, the between-reel variability is about 4 times greater than the within-reel variability indicating that greater differences between reel averages exist than would be expected on the basis of the within-reel variability. In one case, the above suggests that 95% of the reel averages would fall within ± 0.382 pounds of the average and in the other ± 1.664 pounds of the average. The frequency distribution in Table II indicates that only about 30 of 62 reel averages fell within the closer limits. On the other hand only about 2 of the reel averages fell outside the wider limits. Roughly, the same situation exists for the other mills.

From this standpoint, the estimates of between-reel variability give a better characterization of the dispersion of the reel averages. It may be noted, however, that the mere fact that the two estimates do not coincide is an indication that departure from the ideal statistical conditions are occurring. Under these circumstances, considerable care must be exercised in the use of these statistics.

It may also be noted that the between-reel variability might also have been expressed in terms of standard deviation. This might then be compared with the standard deviation based on the within reel tests. Because of the relatively large differences between reels, a standard deviation calculated from the between-reel standards error would be expected to give a highly inflated estimate of the variability within reels. For example, for Mill A it might be expected that 95% of the individual weight readings in a reel would be within ± 2 (0.662) or ± 1.324 lb. of the reel average based on the actual within-reel variability. The corresponding figure for between reel variability would be 4.35 times 1.324 or ± 5.77 lb.—— an unrealistically high figure.

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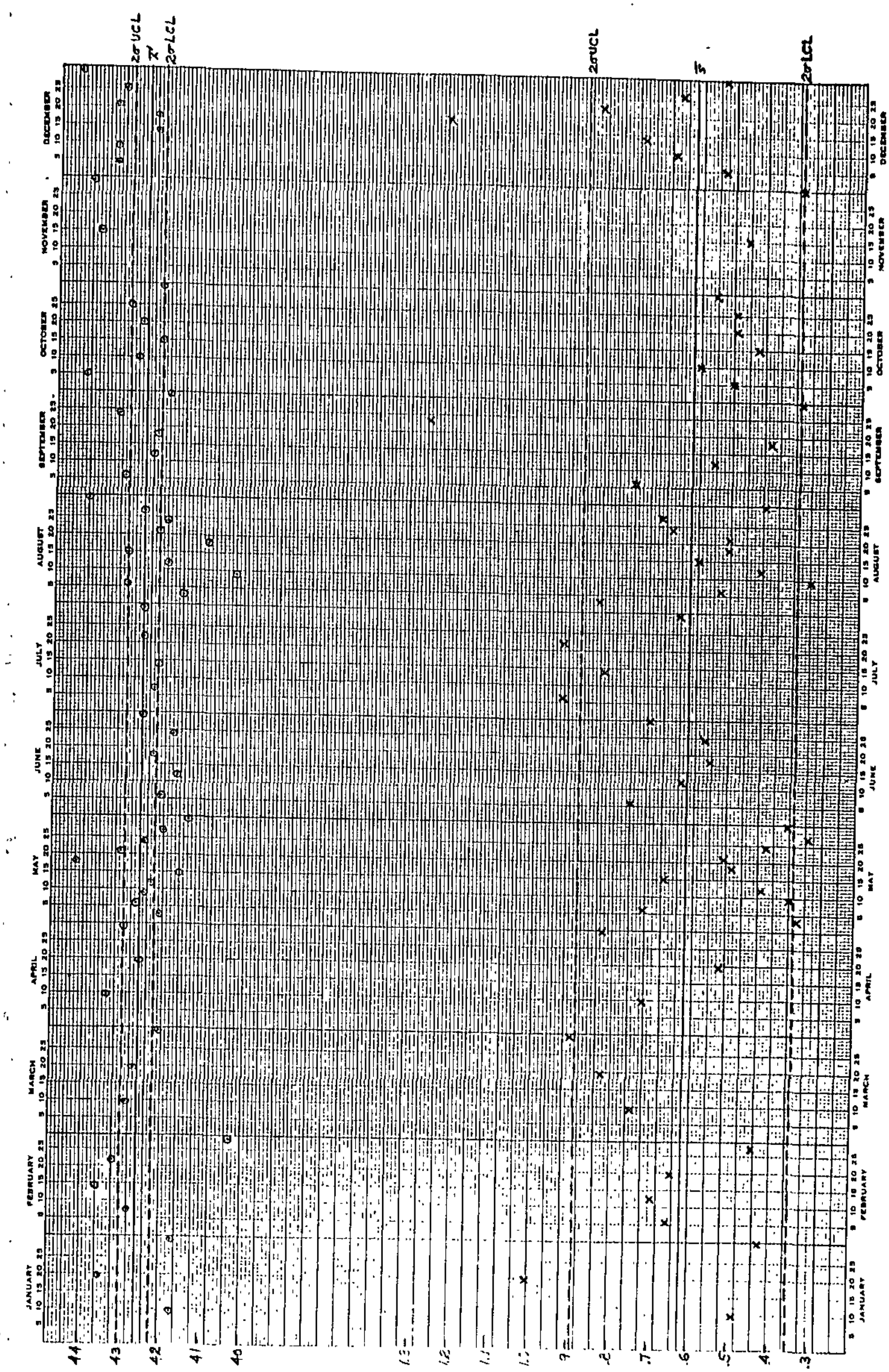

Figure 2

Basis Weight—Mill A

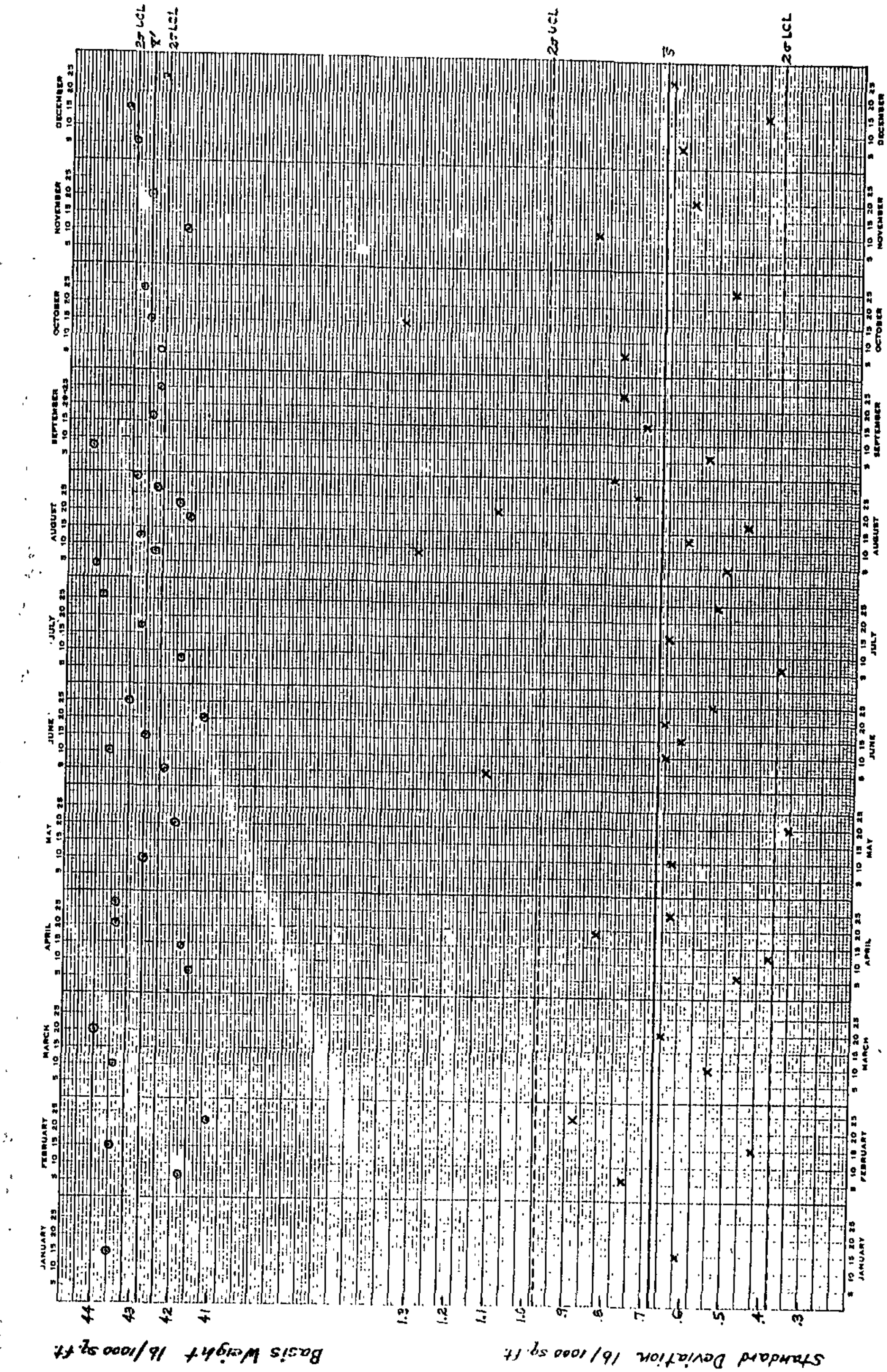

Figure 3

Basis Weight—Mill B

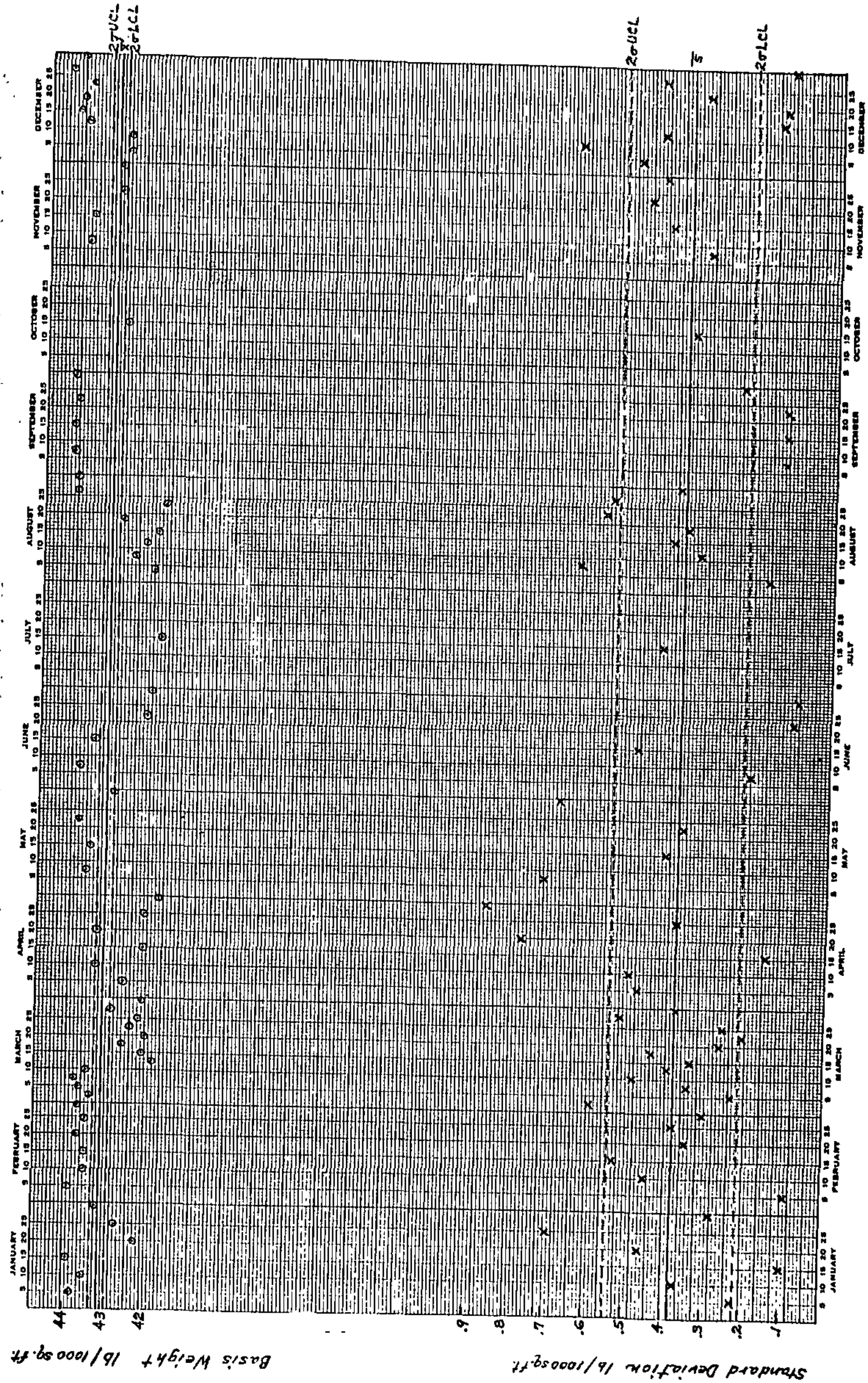

Figure 4

Basis Weight--Mill C

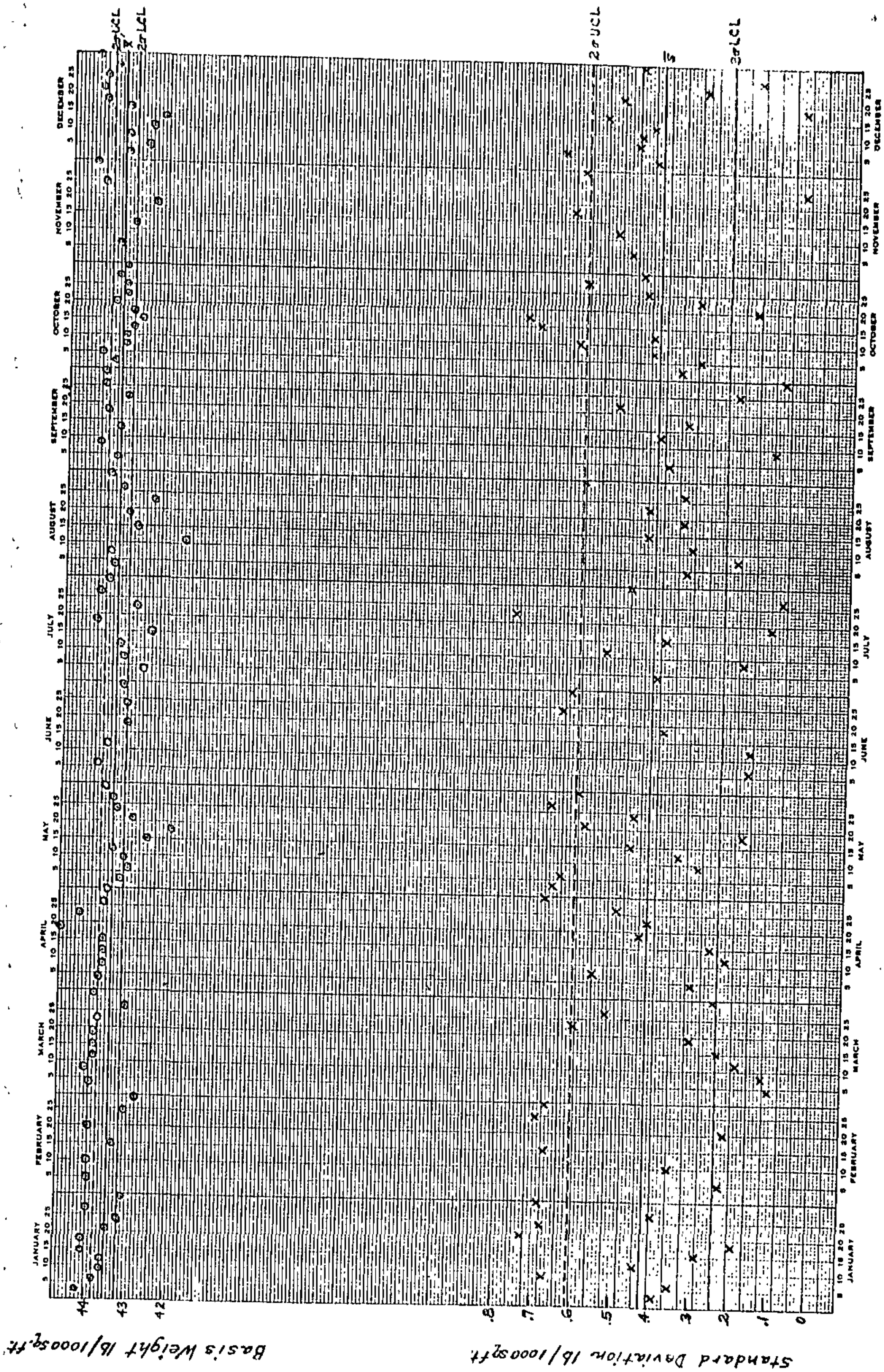

Figure 5

Basis Weight--Mill D

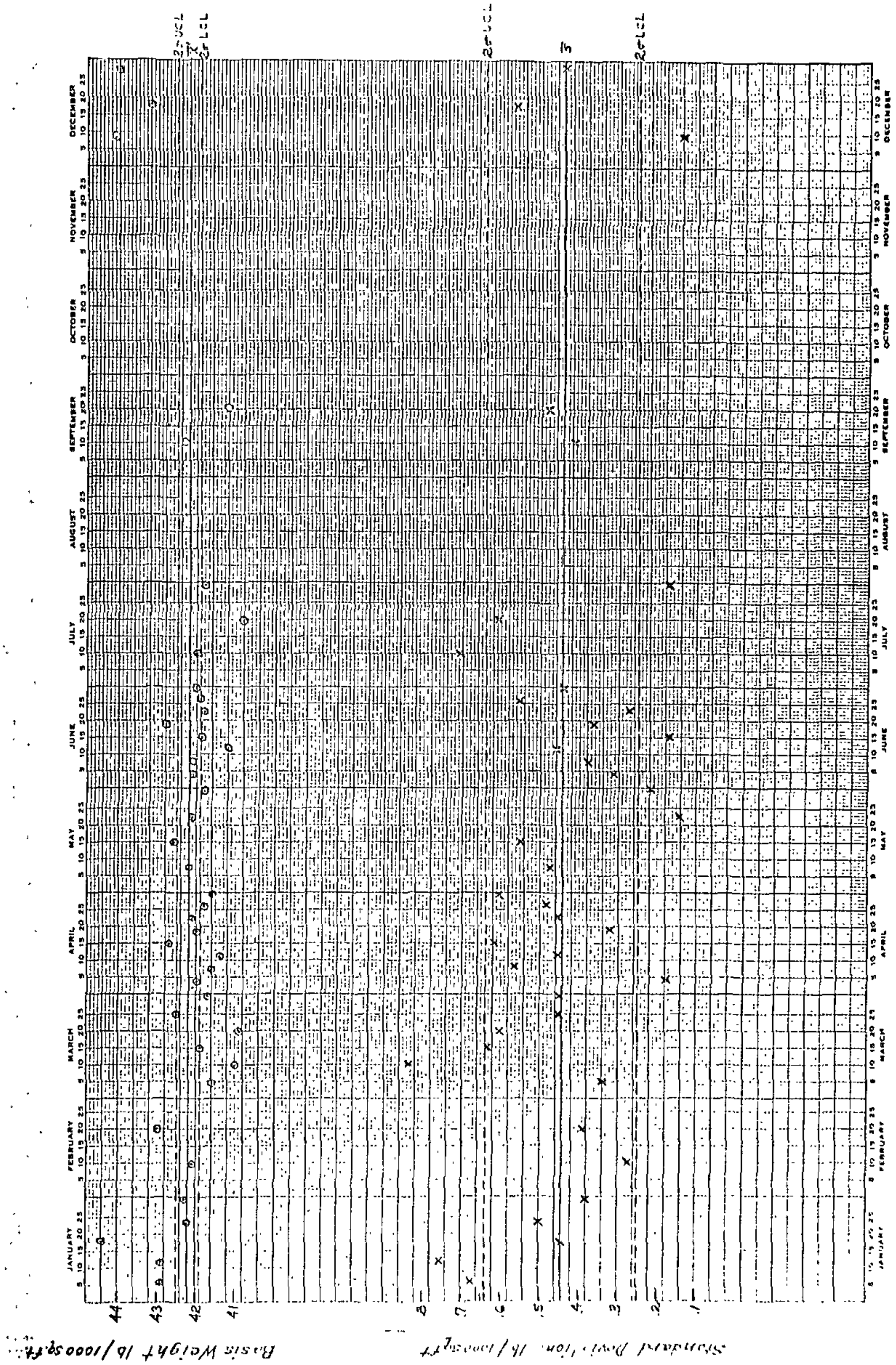

Figure 6

Basis Weight--Mill E

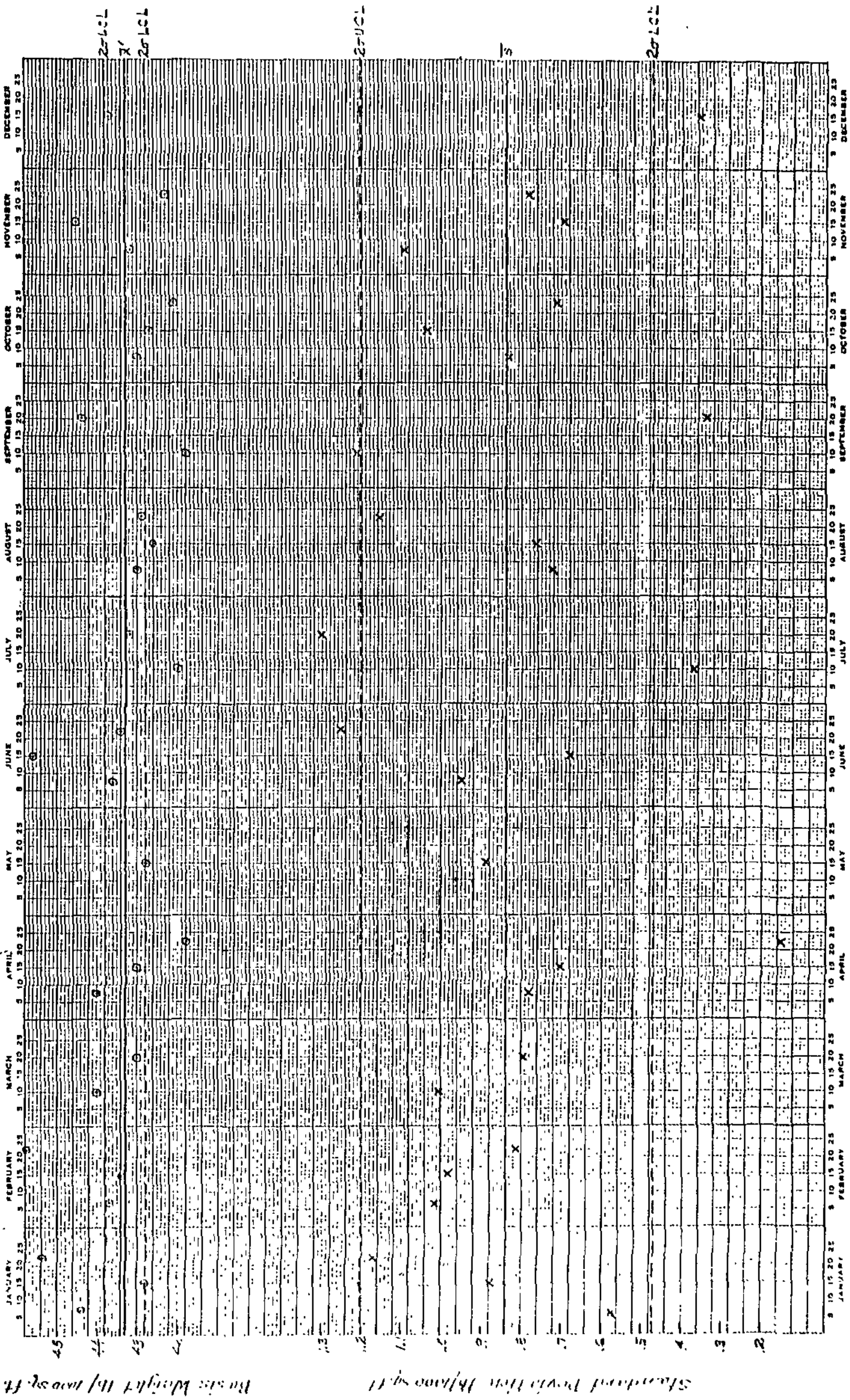

Figure 7

Basis Weight—Mill F

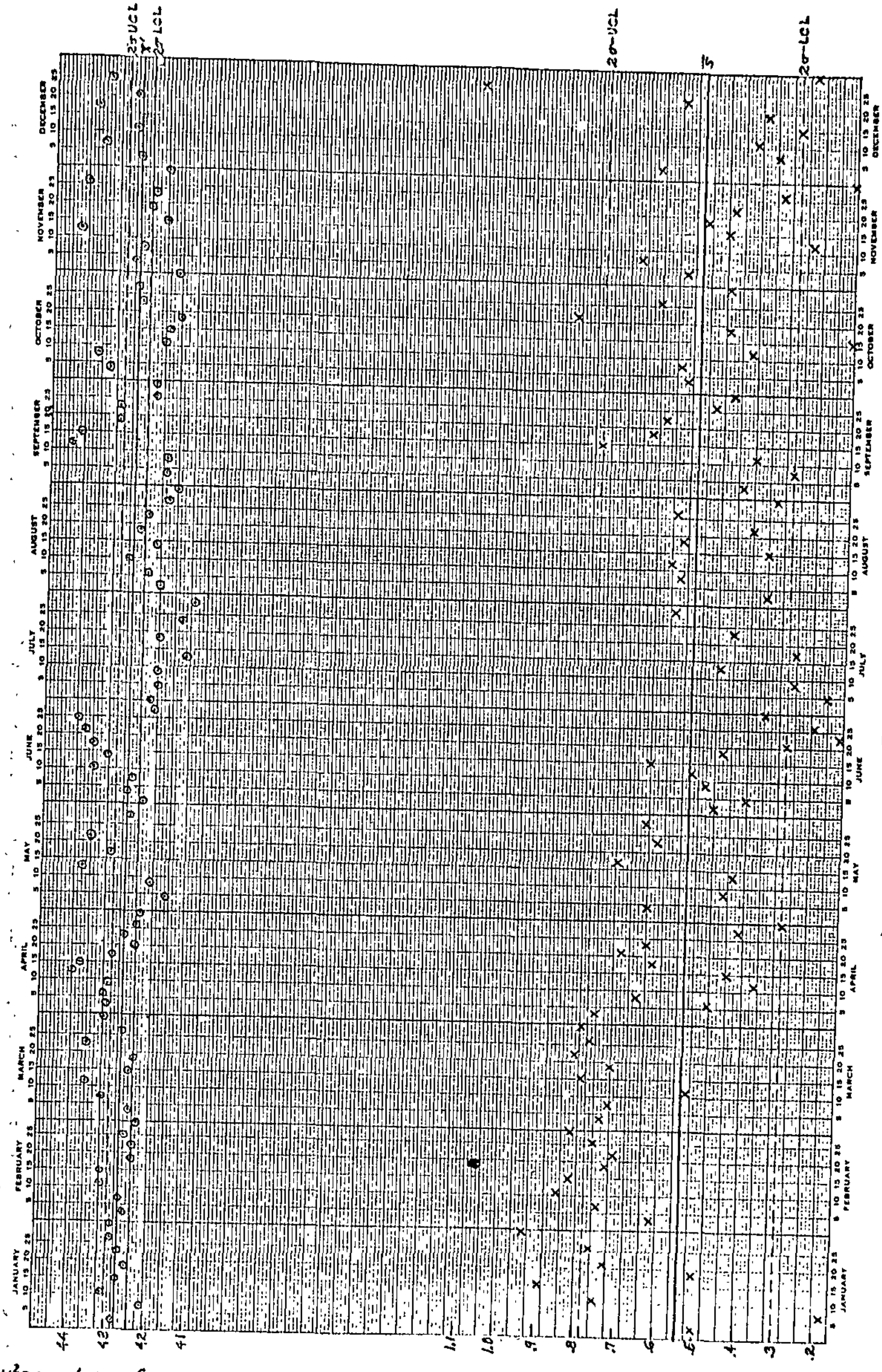

Figure 8

Basis Weight--Mill G

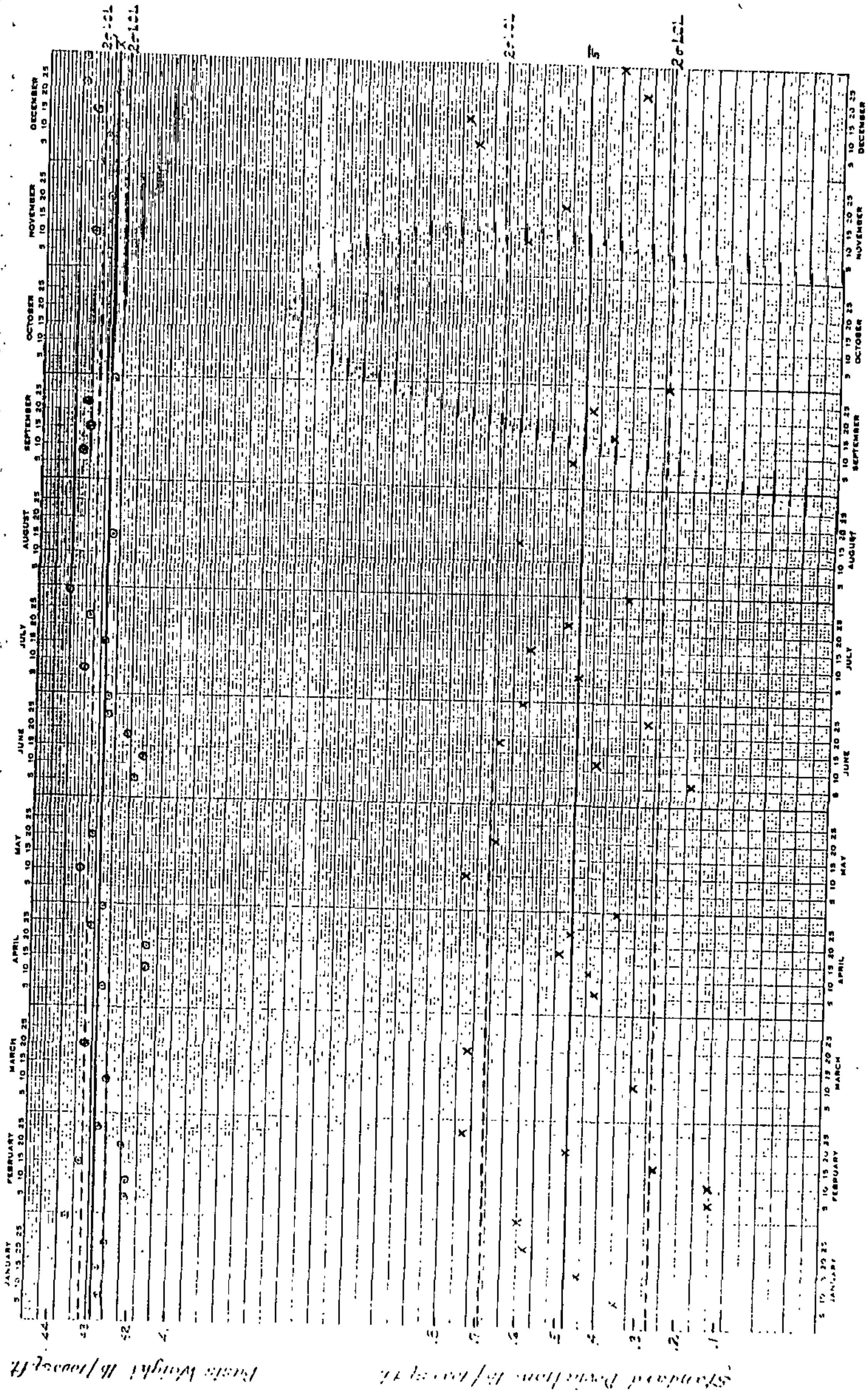

Figure 9

Basis Weight--Mill H

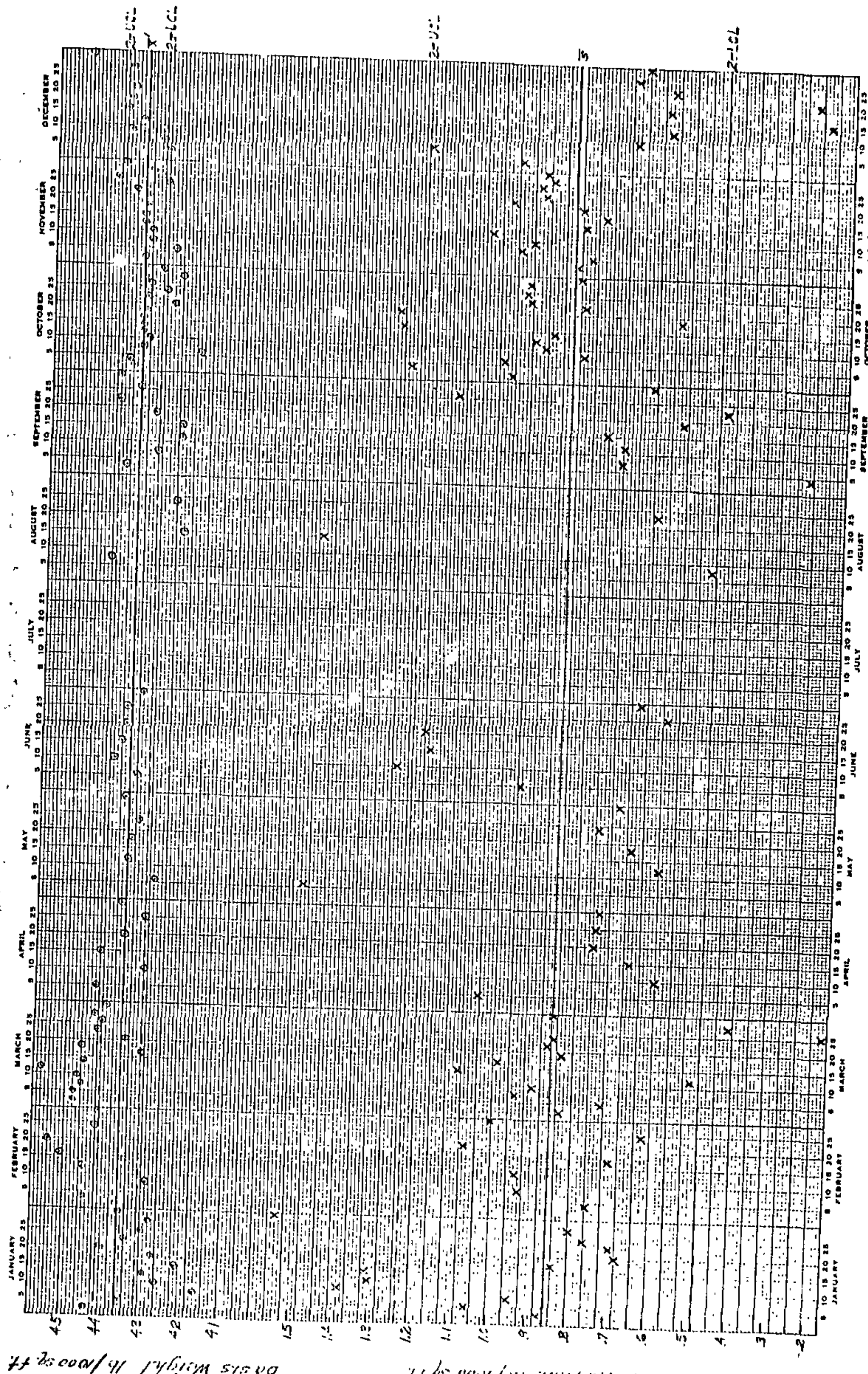

Figure 10

Basis Weight--Mill I

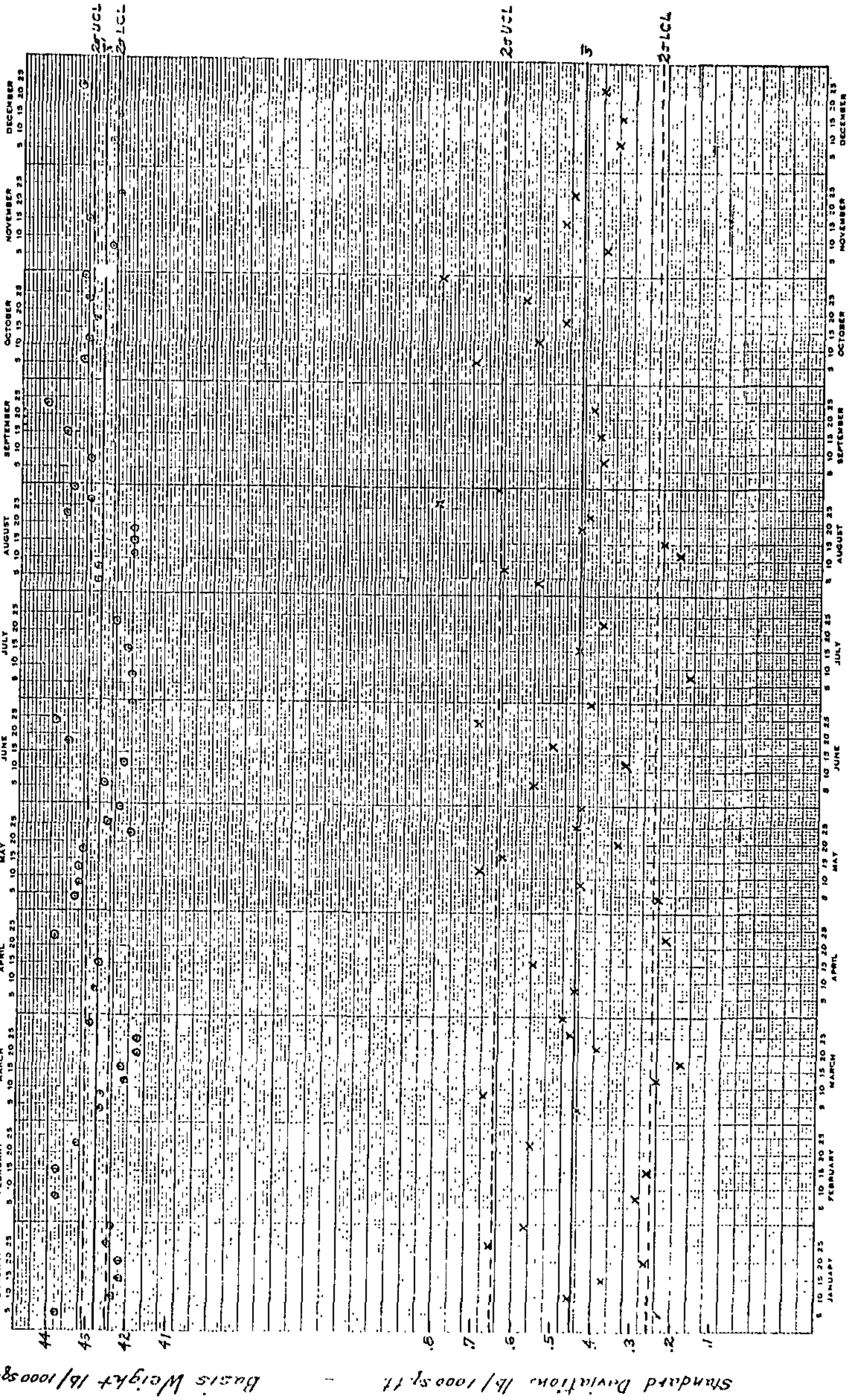

Figure 11
Basis Weight--Mill J

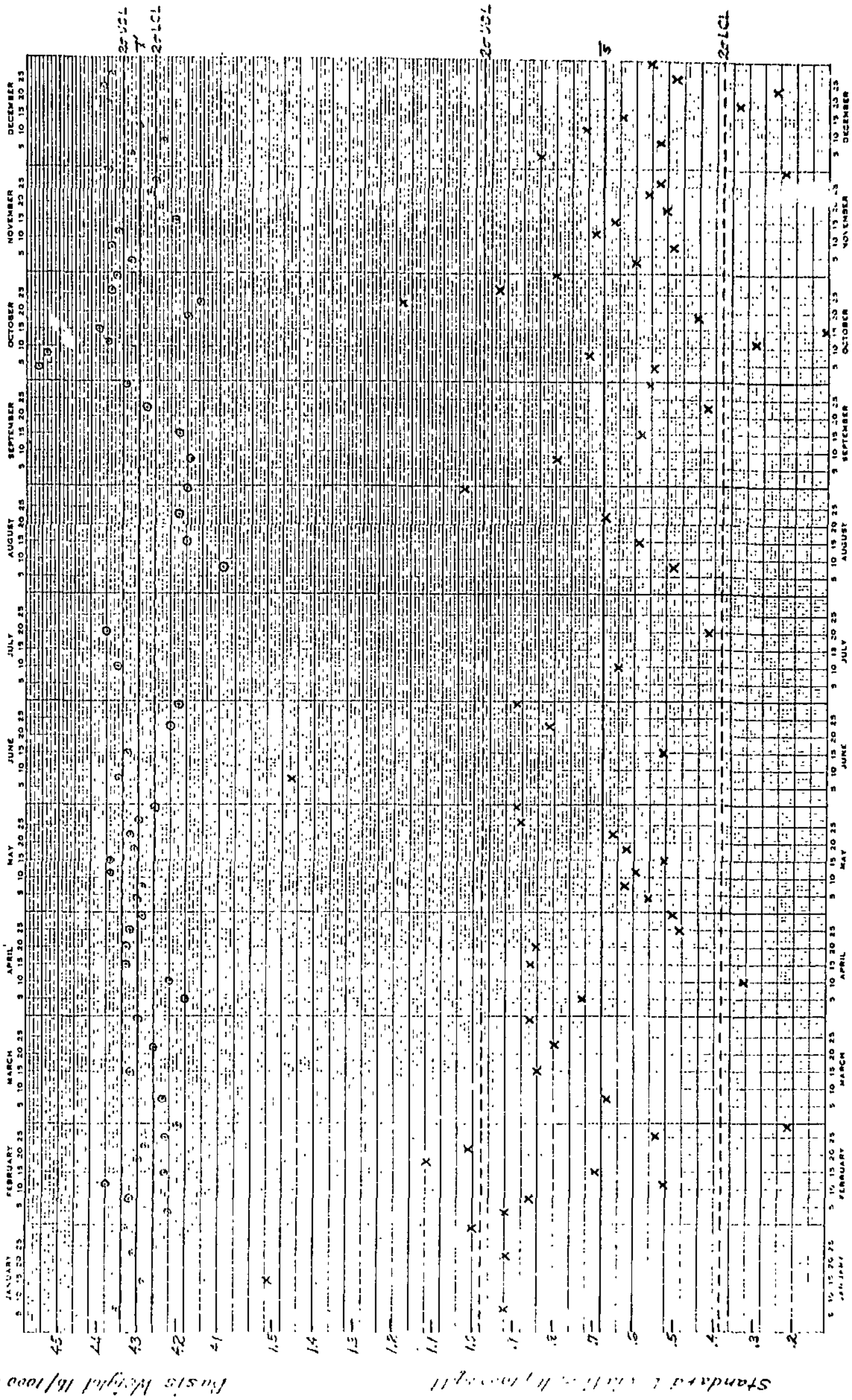

Figure 12

Basis Weight--Mill K

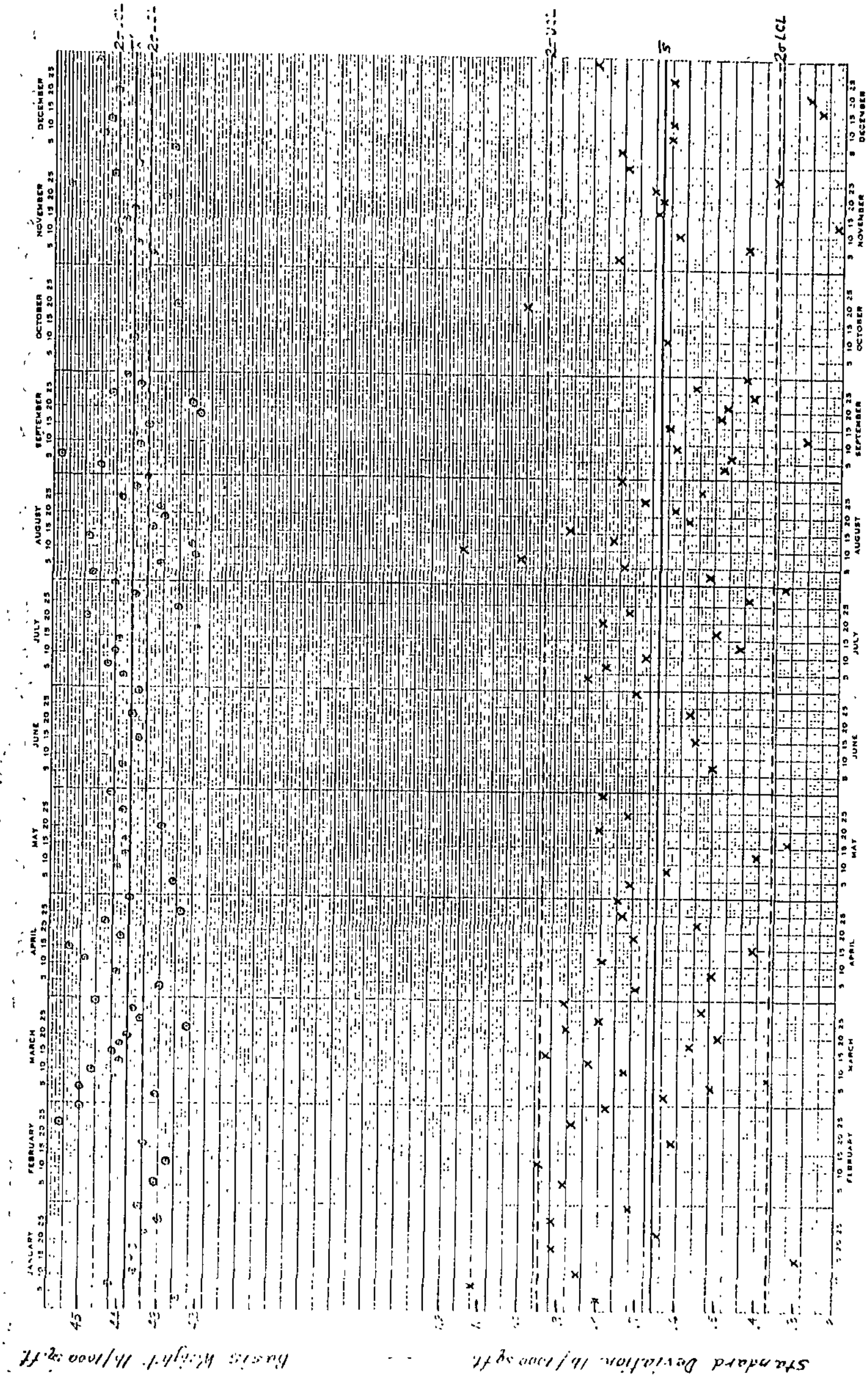

Figure 13

Basis Weight—Mill L

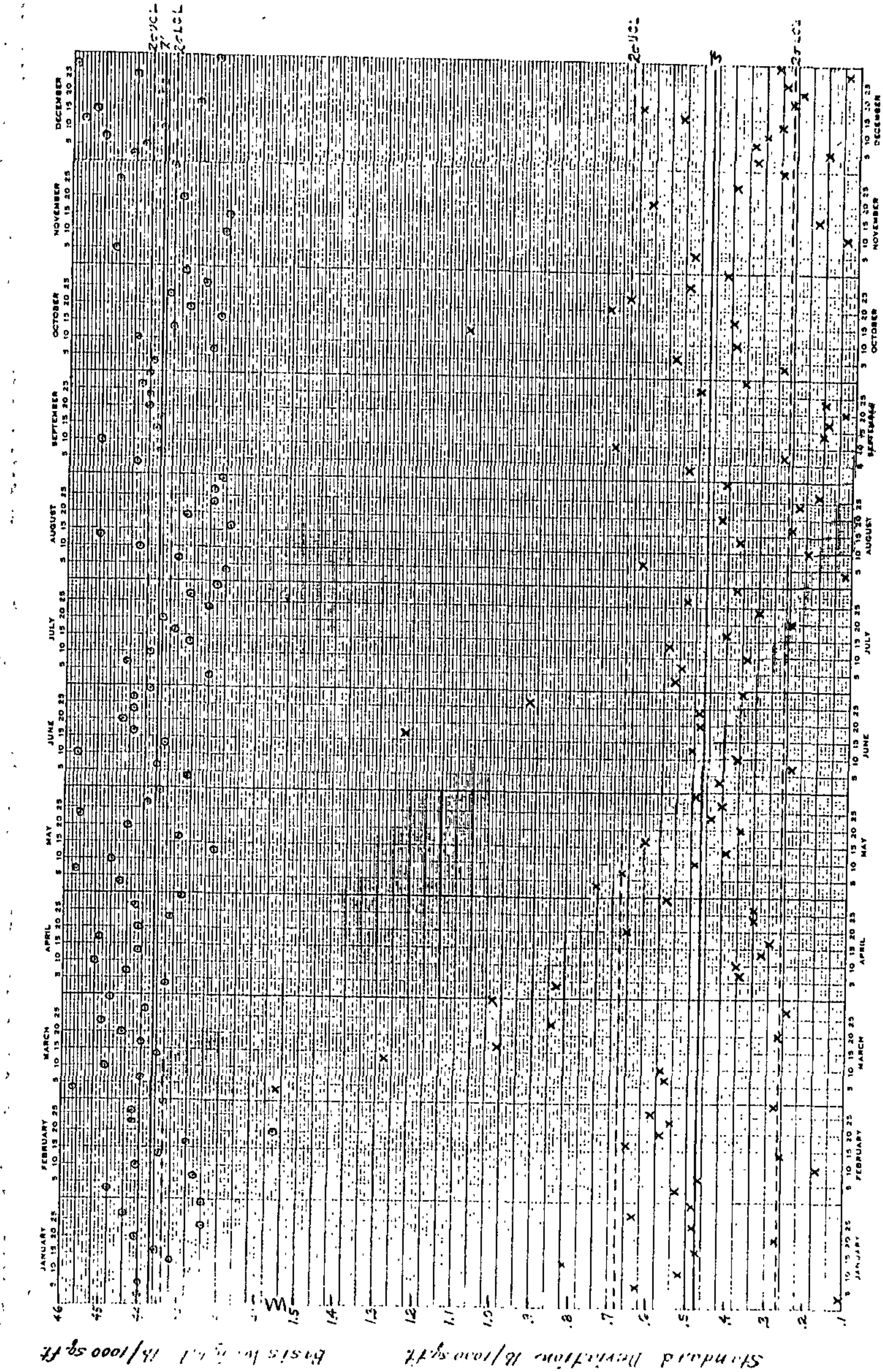

Figure 14

Basis Weight--Mill M

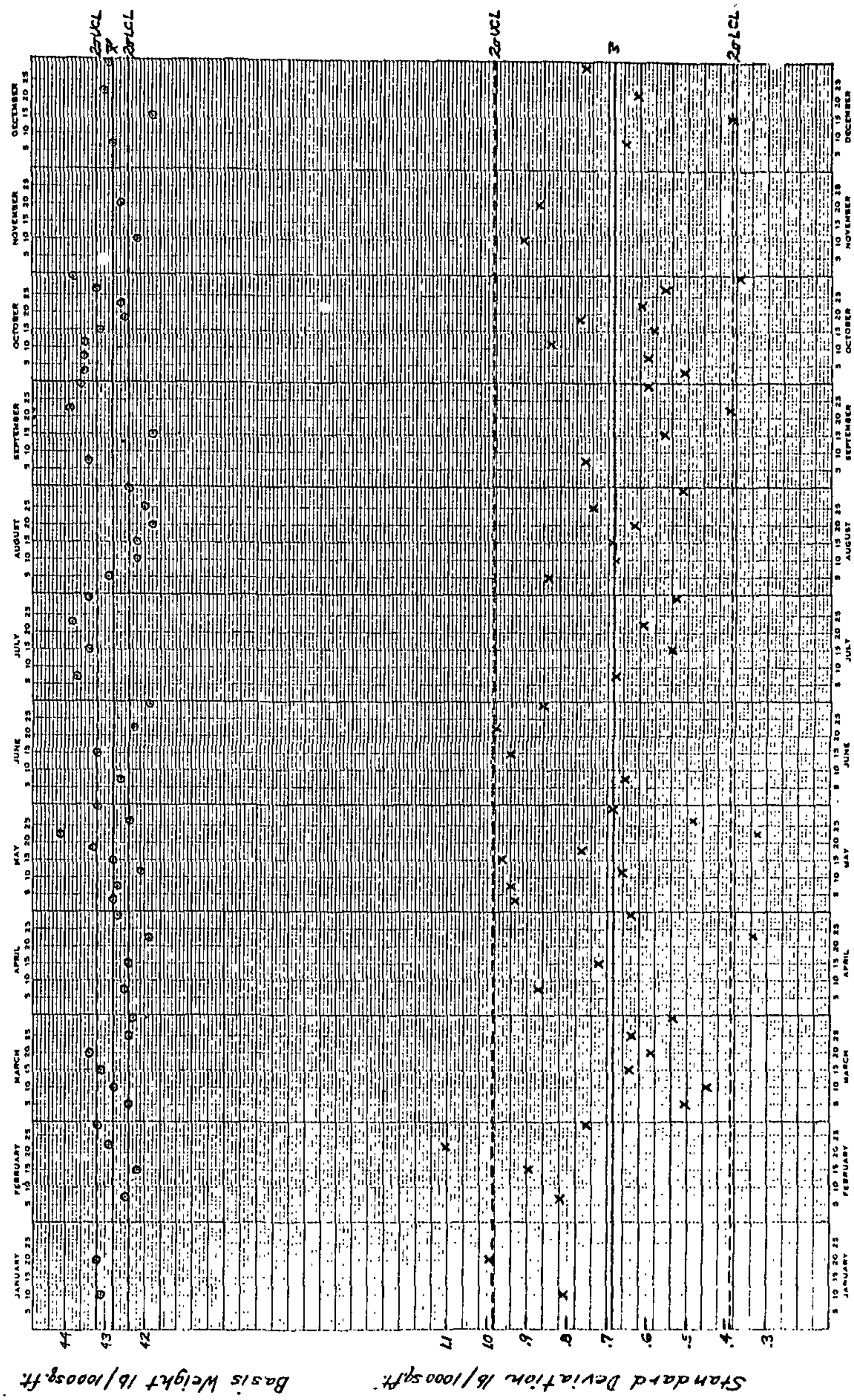

Figure 15

Basis Weight--Mill N

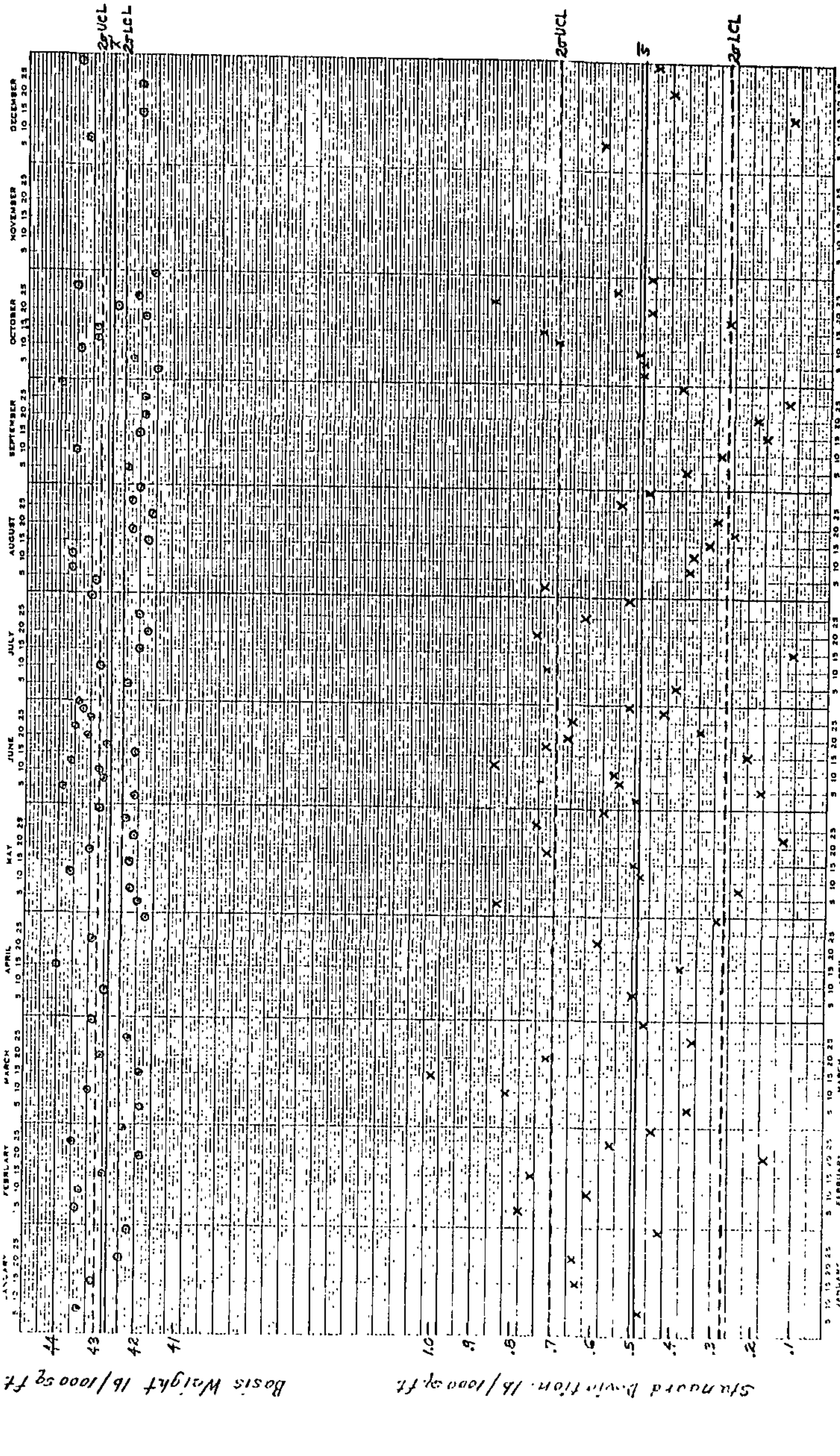

Figure 15

Basis Weight--Mill O

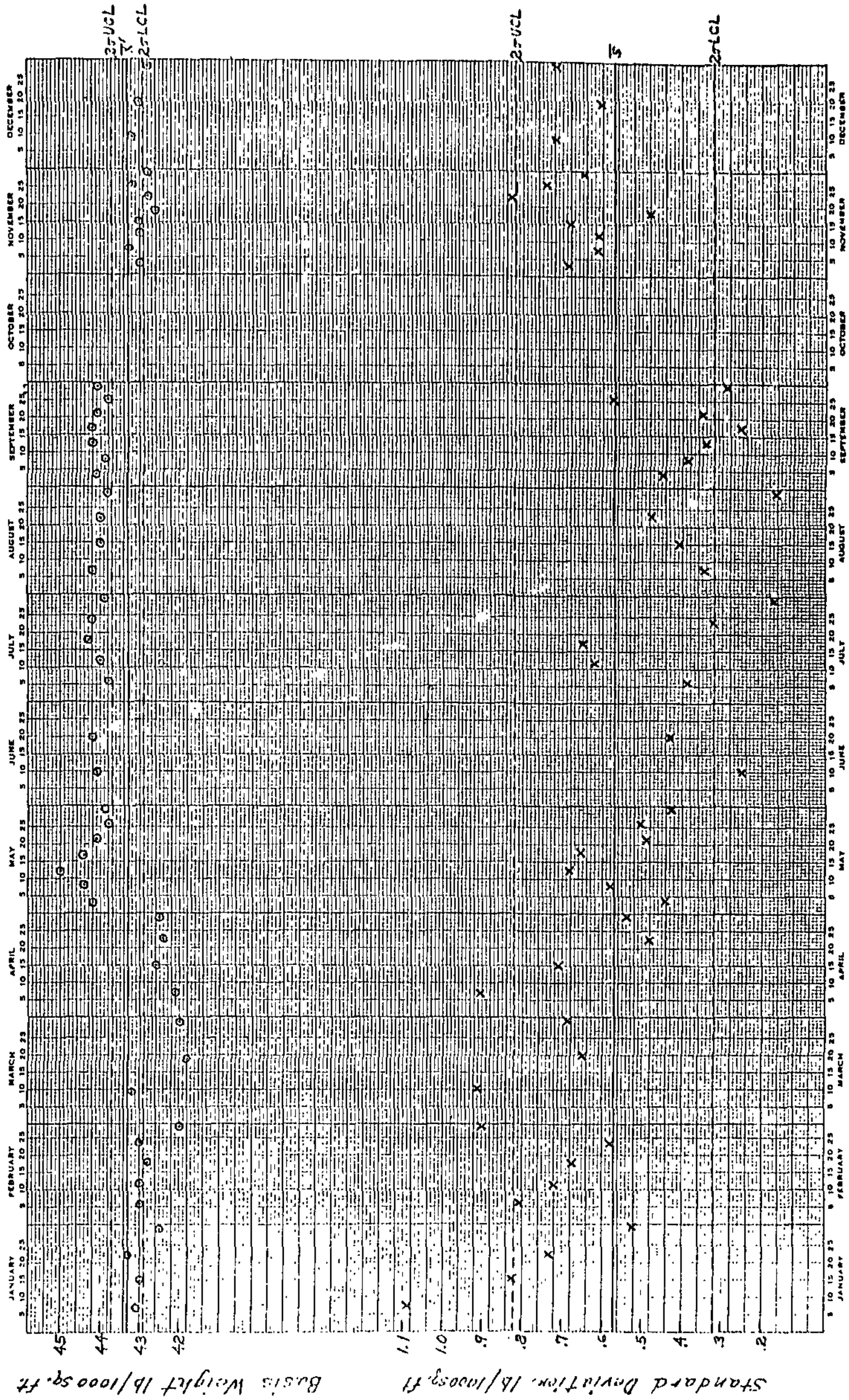

Figure 17

Basis Weight--Mill P

high reel averages and two sets of low reel averages. Mill C exhibited a similar behavior.

The standard deviations (within reel variability) for Mill P appear to be almost negatively correlated with the trends in average discussed above. For example, during the early portion of the year when the reel averages were relatively low, the sample standard deviations which reflect cross-machine variability were relatively high. In the summer months when high reel averages were obtained, the sample standard deviations were relatively low. In the last two months of the year when the reel averages were lowered to about the level prevailing at the start of the year, the standard deviations increased to about the same level obtained at the start of the year. The control chart can supply no reason or cause for the above behavior; however, its helpfulness in assessing the significance of such changes may be important.

Considering the control charts for weight as a whole, it appears that substantial shifts in reel averages can and do occur over various periods of time for most mills. As noted previously, it is not suggested that control of mill quality within limits based on the cross-machine variability is either necessary or desirable. On the other hand, limits based on the within reel variance might be looked upon as a potential ideal and, in some cases, at least certain mills tend to approach this situation. For example, for Mill P, if the average and limits during the summer months were shifted upwards in line with the actual reel averages, the reel averages would appear to be in statistical control.

With regard to sample standard deviations, the impression persists that sample standard deviations (within reel) tend to approach being in statistical control more closely than do the reel averages. This would seem to indicate that the cross-machine variability of most machines tends to remain more constant than the reel averages.

CALIPER

The comparisons of within and between reel variability for caliper are summarized in Table III. Referring to the table, it may be noted that Mill E exhibited the greatest within reel variability--5.78% 2 standard deviation--while mill C exhibited the smallest within reel variability--3.41%. Comparing the within-reel with the between-reel variability (both on a per cent two standard error basis), it may be noted that between-reel variability was usually five or six times greater than the within-reel variability with Mill E exhibiting the greatest (9.0%) and Mill O the least (2.21%).

As would be expected from the above, the frequency distribution of the reel averages shown in Table IV for each mill indicate the reel average for caliper spread over a relatively large range. Multi-peaked distribution are relatively common, e.g., see Mills B, D, G, J, K, M, etc. An extreme case of a skewed distribution may be noted for Mill P.

Control charts for each mill are shown in Figures 18 through 33 inclusive. Because the between reel variance was considerably greater than the within-reel variance, the control charts in many respects are quite similar to the weight charts previously discussed. Thus the small within reel variance resulted in extremely close control limits on the reel averages with

TABLE III

COMPARISON OF WITHIN AND BETWEEN REEL VARIABILITY FOR CALIPER

Mill	A	B	C	D	E	F	G	H	I
Within Reel									
No. of samples	62	38	64	98	41	29	96	38	98
Grand av., $\bar{X}'$	13.5	12.7	12.6	12.5	12.0	13.1	12.4	13.3	12.1
Av. standard deviation, $\bar{s}$	0.308	0.342	0.208	0.244	0.336	0.314	0.298	0.321	0.250
Estimated population, standard deviation, σ	0.318	0.353	0.215	0.252	0.347	0.324	0.308	0.331	0.258
Per cent Two standard deviation	4.71	5.56	3.41	4.03	5.78	4.95	4.97	4.97	4.26
Two st. error, $2\sigma/\sqrt{n}$	0.130	0.144	0.098	0.102	0.142	0.132	0.126	0.136	0.106
Per cent two standard error	0.96	1.13	0.78	0.82	1.18	1.01	1.02	1.02	0.88
Two SE limits about $\bar{X}'$	13.4-13.6	12.6-12.8	12.5-12.7	12.4-12.6	11.9-12.1	13.0-13.2	12.3-12.5	13.2-13.4	12.0-12.2
Two SE limits about $\bar{s}$	0.201-0.415	0.240-0.444	0.146-0.270	0.171-0.317	0.236-0.436	0.220-0.408	0.209-0.387	0.225-0.417	0.176-0.324
Between Reels									
Two standard error, $2\sigma_{\bar{x}}/\sqrt{N}$	0.718	0.836	0.964	0.540	1.080	0.896	0.778	0.572	0.586
Per cent two standard error	5.32	6.58	7.65	4.32	9.00	6.84	6.23	4.30	4.84

Mill	J	K	L	M	N	O	P	Q	Composite
Within Reel									
No. of samples	56	68	92	108	56	74	52	6	1076
Grand av., $\bar{X}'$	12.5	12.3	12.7	12.8	13.2	13.4	12.8	11.3	12.7
Av. standard deviation, $\bar{s}$	0.250	0.297	0.303	0.239	0.246	0.336	0.266	0.195	0.284
Estimated population, standard deviation, σ	0.258	0.307	0.313	0.247	0.254	0.347	0.375	0.201	0.293
Per cent Two standard deviation	4.13	4.99	4.93	3.86	3.85	5.18	4.30	3.56	4.61
Two st. error, $2\sigma/\sqrt{n}$	0.106	0.126	0.128	0.100	0.104	0.142	0.112	0.082	0.120
Per cent two standard error	0.85	1.02	1.01	0.78	0.79	1.06	0.88	0.73	0.94
Two SE limits about $\bar{X}'$	12.4-12.6	12.2-12.4	12.6-12.8	12.7-12.9	13.1-13.3	13.3-13.5	12.7-12.9	11.2-11.4	12.6-12.8
Two SE limits about $\bar{s}$	0.176-0.324	0.208-0.386	0.213-0.393	0.168-0.310	0.173-0.319	0.236-0.436	0.187-0.345	0.137-0.253	0.199-0.369
Between Reels									
Two standard error, $2\sigma_{\bar{x}}/\sqrt{N}$	0.826	0.804	0.800	0.842	0.648	0.296	0.830	0.570	0.744
Per cent two standard error	6.61	6.54	6.30	6.58	4.92	2.21	6.88	5.04	5.86

TABLE IV

FREQUENCY DISTRIBUTION OF CALIPER AVERAGES

Caliper, points	A	B	C	D	E	F	G	H	I	J	K (Mills)	L	M	N	O	P	Q	Total	Per Cent	Cumulative, Total	Cumulative, %
14.6															1			1	0.1	1	0.1
14.5																					
14.4																					
14.3															1			1	0.1	2	0.2
14.2	2														2			4	0.4	6	0.6
14.1	2														2			4	0.4	10	0.9
14.0	2													2	4			8	0.7	18	1.7
13.9	4					2									5			11	1.0	29	2.7
13.8	8							2						2	4			16	1.5	45	4.2
13.7	8					1		2			2		1	3	3			20	1.9	65	6.0
13.6	3	1	1			1		4				2		3	3			18	1.7	83	7.7
13.5	6		4			2		3					4	2	9			30	2.8	113	10.5
13.4	3	2	3			2		5					1	5	7			28	2.6	141	13.1
13.3	8	2	2			2	1	4				2	5	4	4			34	3.2	175	16.3
13.2	7	2	2	1	2	1		4				3	11	12	7	3		60	5.6	235	21.8
13.1	4	2	2			5	2	3		4		6	10	5	5	9		57	5.3	292	27.1
13.0	1	3	2	4		2	4	6		8	1	7	13	6	8	17		82	7.6	374	34.8
12.9	2	5	5	11	1	4	9	3	1	7	2	8	10	7	2	7		84	7.8	458	42.6
12.8	1	1	4	11		1	8	1	1	4		8	13	3	1	1		58	5.4	516	48.0
12.7		2	2	10			7	1		3	2	9	3		1	1		41	3.8	557	51.8
12.6		2	8	9		2	6		3	4	5	6	2	1	1	2		51	4.7	608	56.5
12.5	1	5	5	15	2		6		6	2	6	12	9	1	1			71	6.6	679	63.1
12.4		2	3	12	2	2	5		6	5	4	8	7		3	4		63	5.8	742	69.0
12.3		2	2	10	6		10		7	6	10	3	4			2		62	5.8	804	74.7
12.2		6	5	9	5	2	10		11	1	8	5	5			2		69	6.4	873	81.1
12.1			3	5	2		7		19	1	4	2	3					46	4.3	919	85.4
12.0			3		5		9		15	4	10	2	4					52	4.8	971	90.2
11.9		1	3		2		5		10	3	4	1	2					31	2.9	1002	93.1
11.8			2	1	2		3		5	4	6	2	1			1	1	28	2.6	1030	95.7
11.7					2		4		6		2							14	1.3	1044	97.0
11.6			1						4		2					1		8	0.7	1052	97.8
11.5			2		4				3			1				1	1	12	1.1	1064	98.9
11.4					3											1	1	5	0.5	1069	99.3
11.3									1									1	0.1	1070	99.4
11.2																	1	1	0.1	1071	99.5
11.1																					
11.0																	2	2	0.2	1073	99.7
10.9					1													1	0.1	1074	99.8
10.8					1													1	0.1	1075	99.9
10.7					1													1	0.1	1076	100.0
Total	62	38	64	98	41	29	96	38	98	56	68	92	108	56	74	52	6	1076			

Note: Underlined values are the grand average.

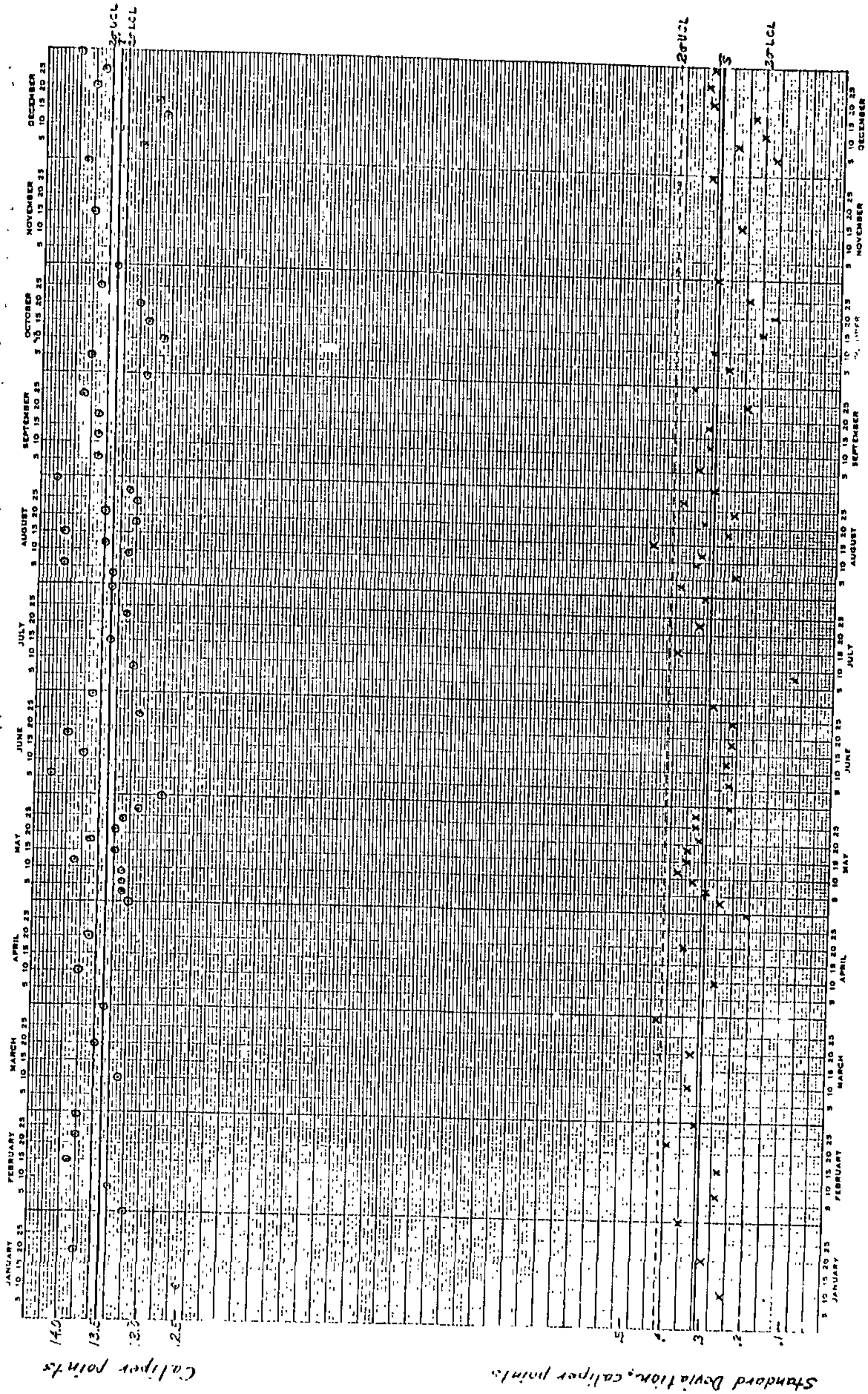

Figure 18

Caliper--Mill A

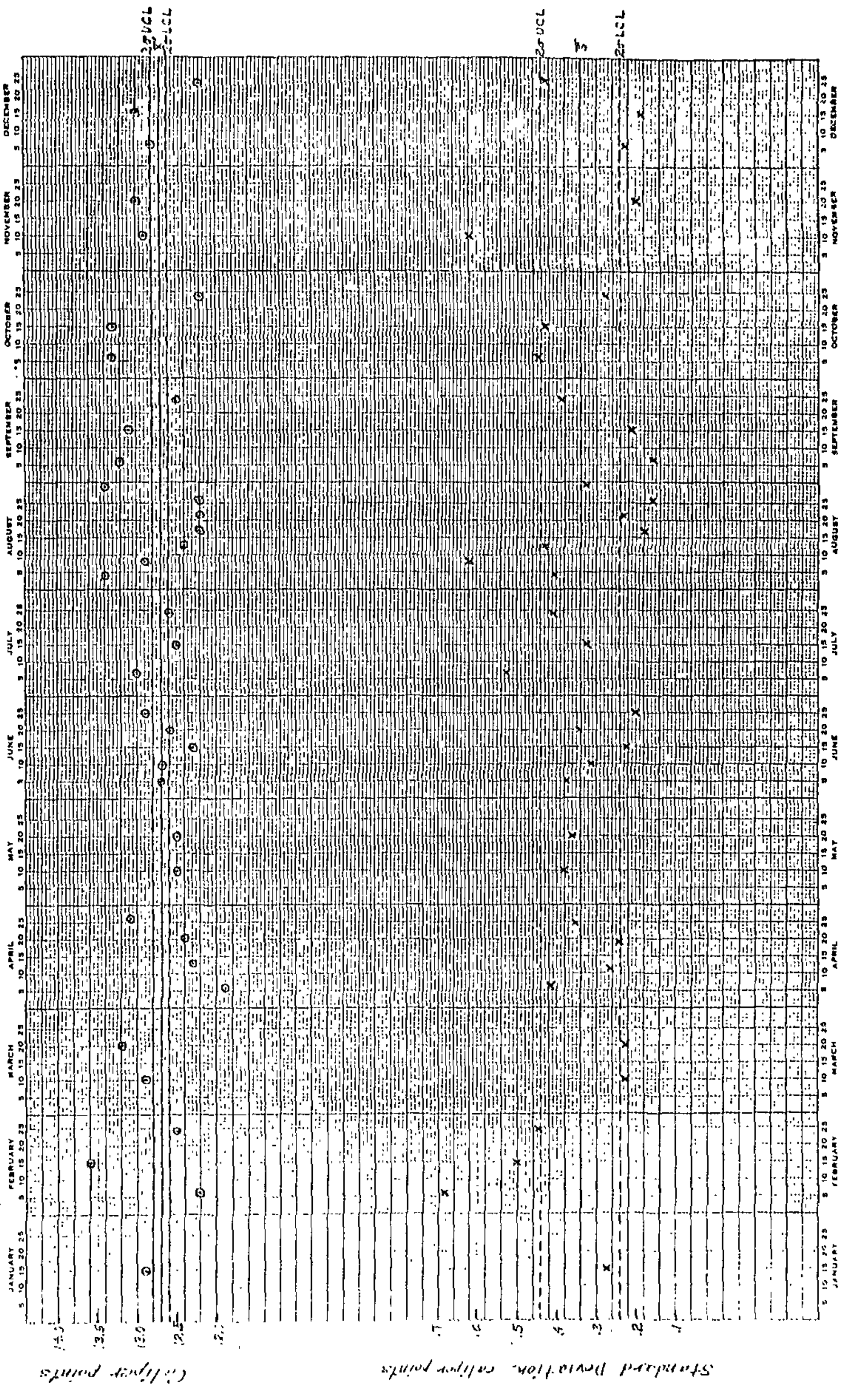

Figure 19

Caliper--Mill B

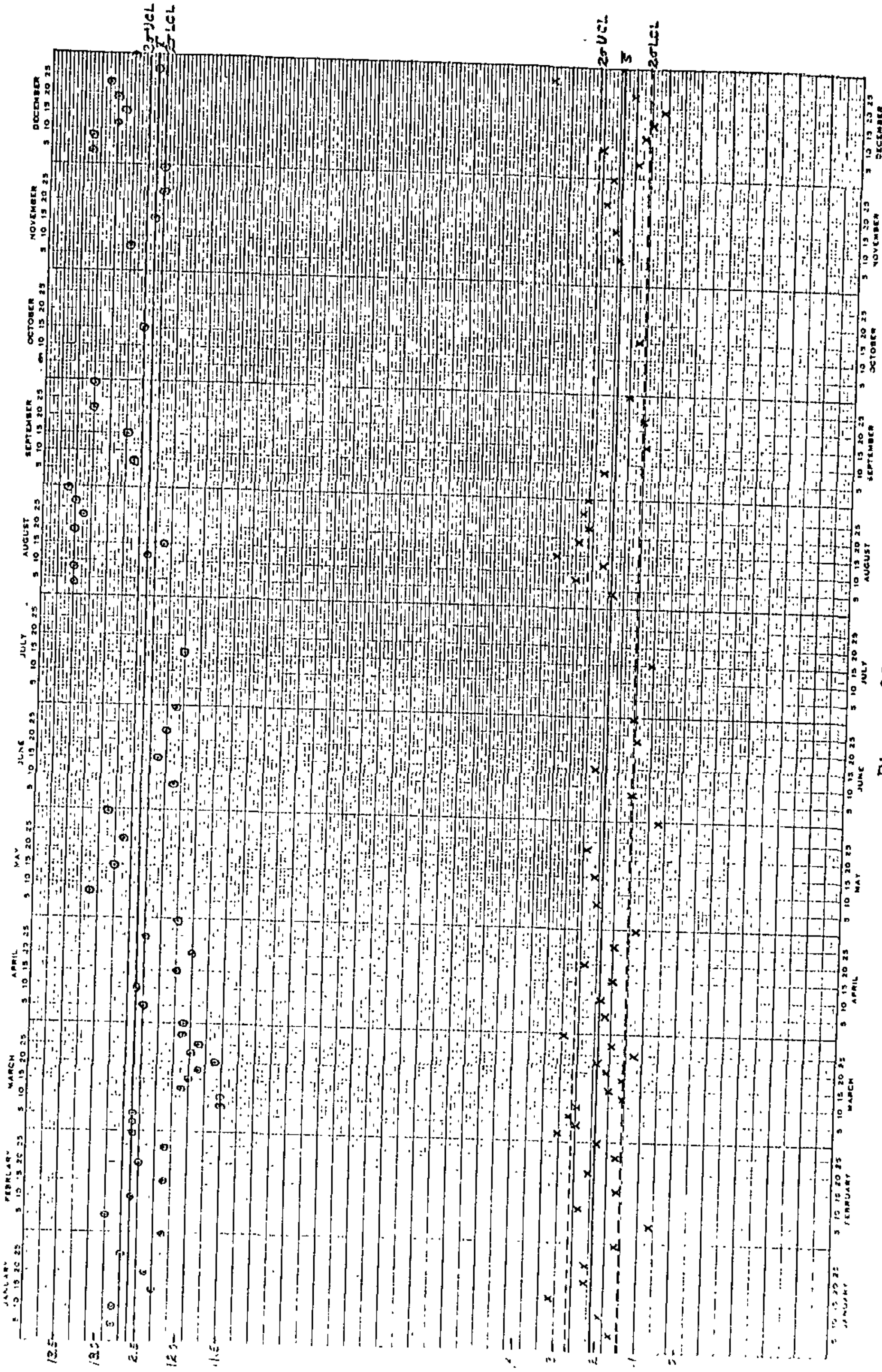

Figure 20

Caliper--Mill C

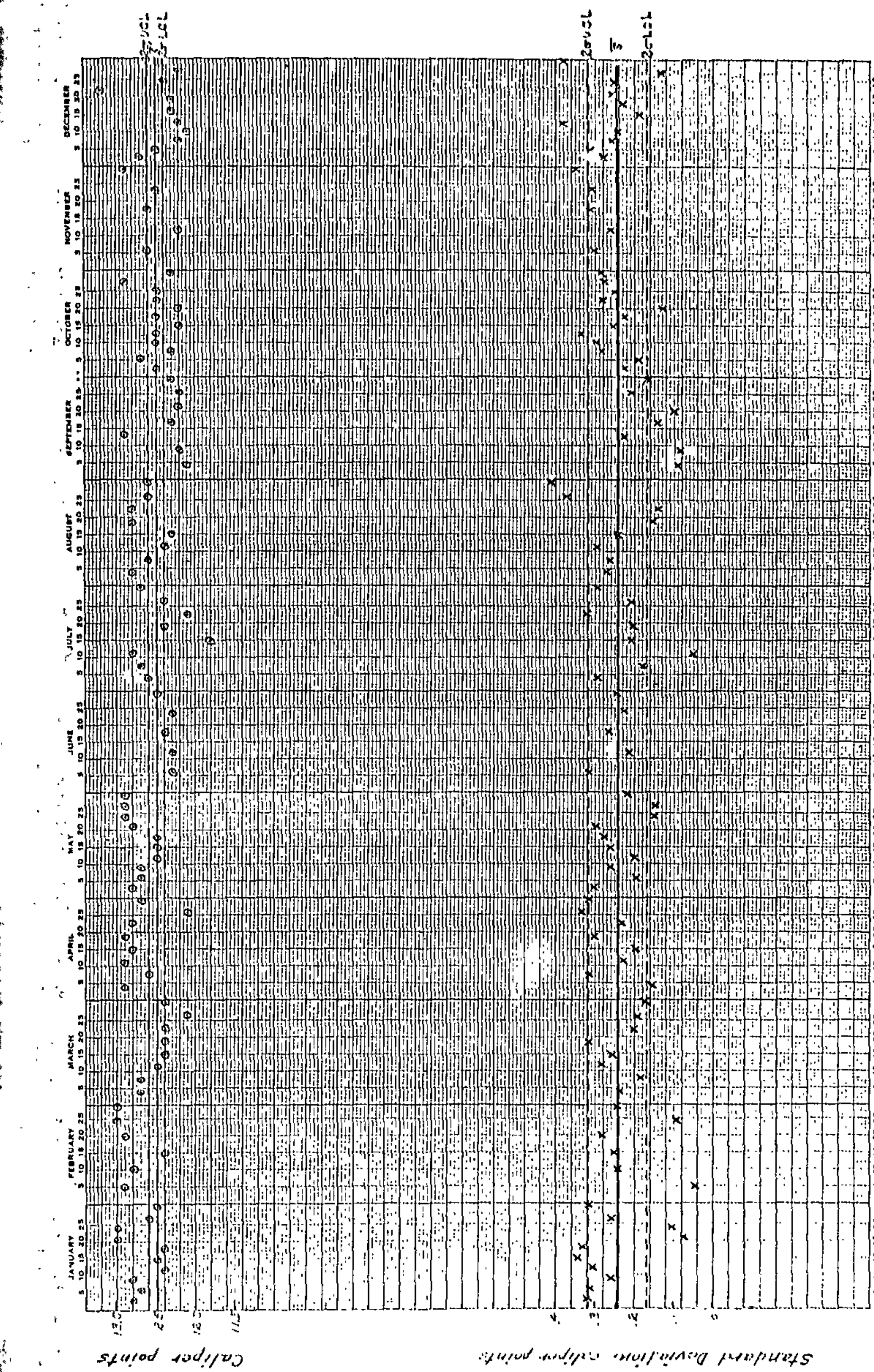

Figure 21

Caliper--Mill D

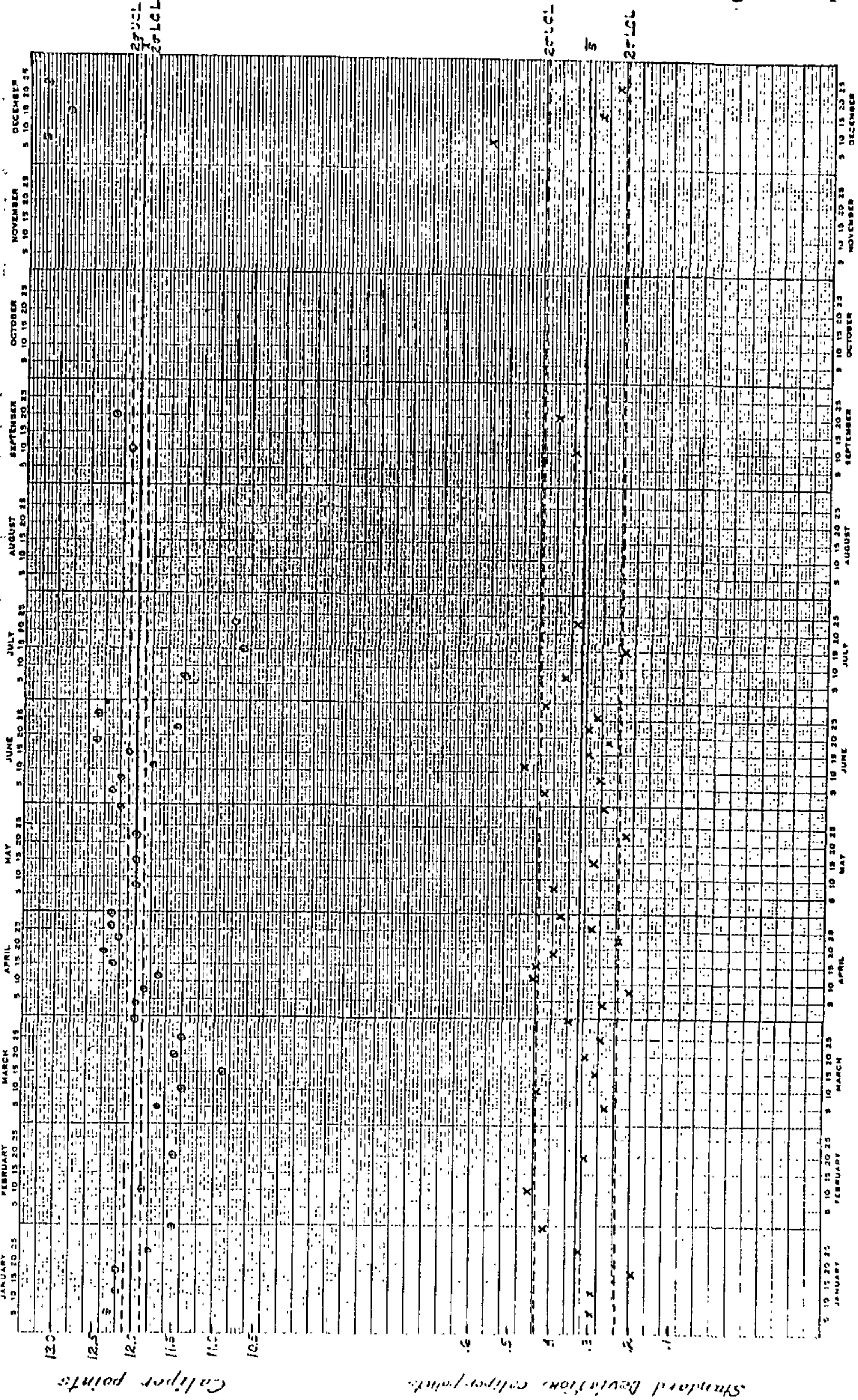

Figure 22

Caliper--Mill E

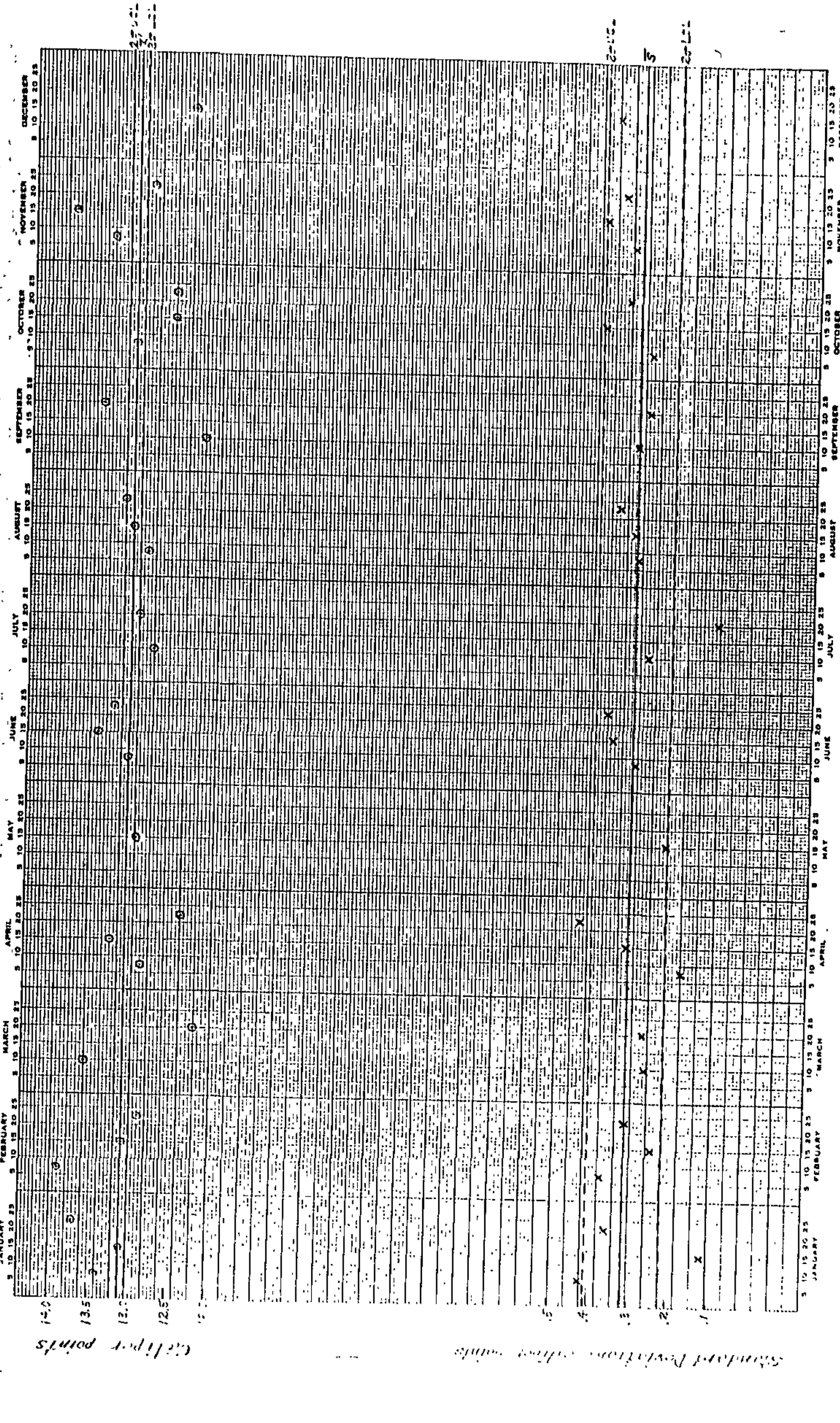

Figure 23

Caliper--Mill F

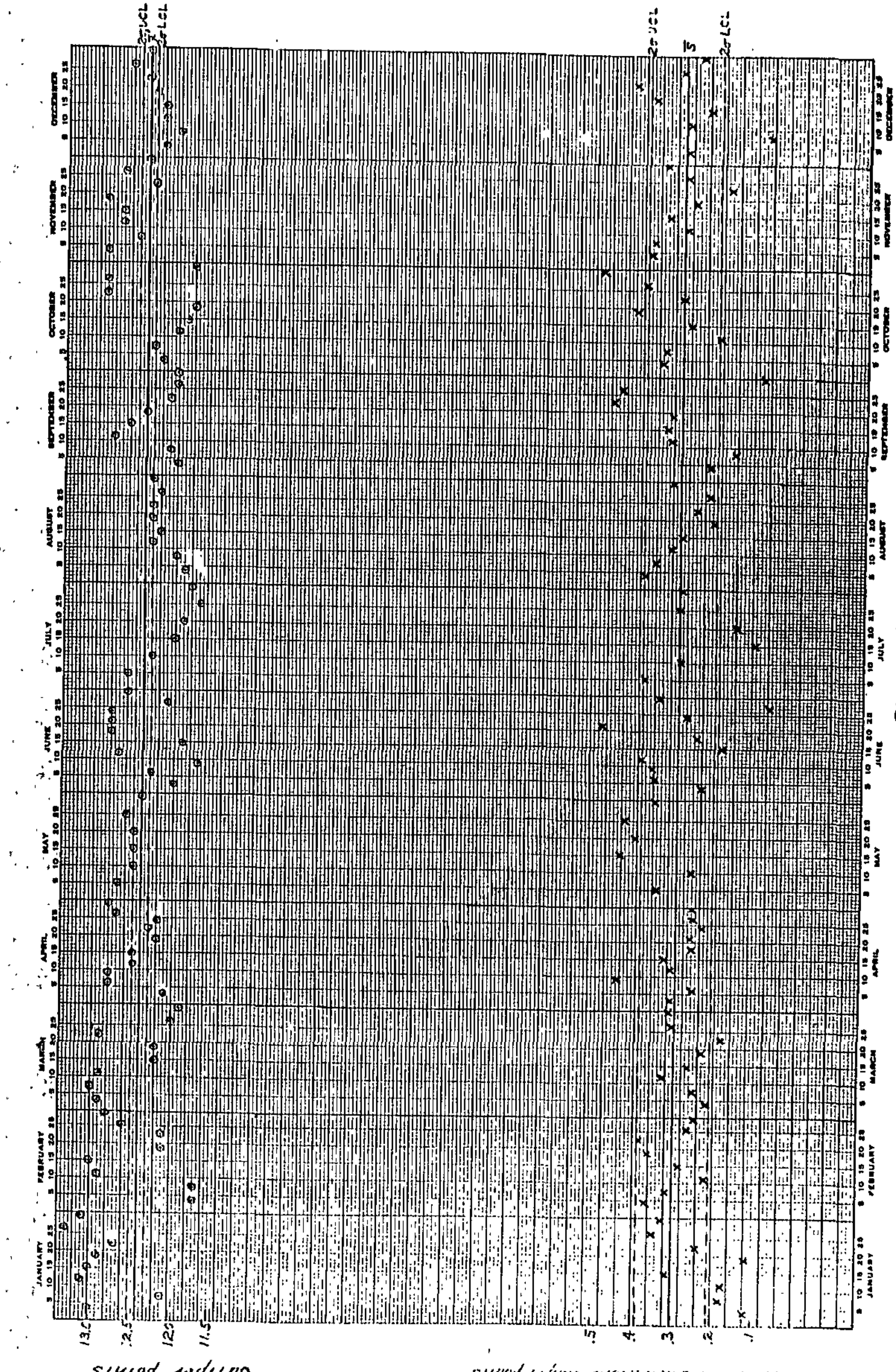

Figure 24

Caliper--Mill G

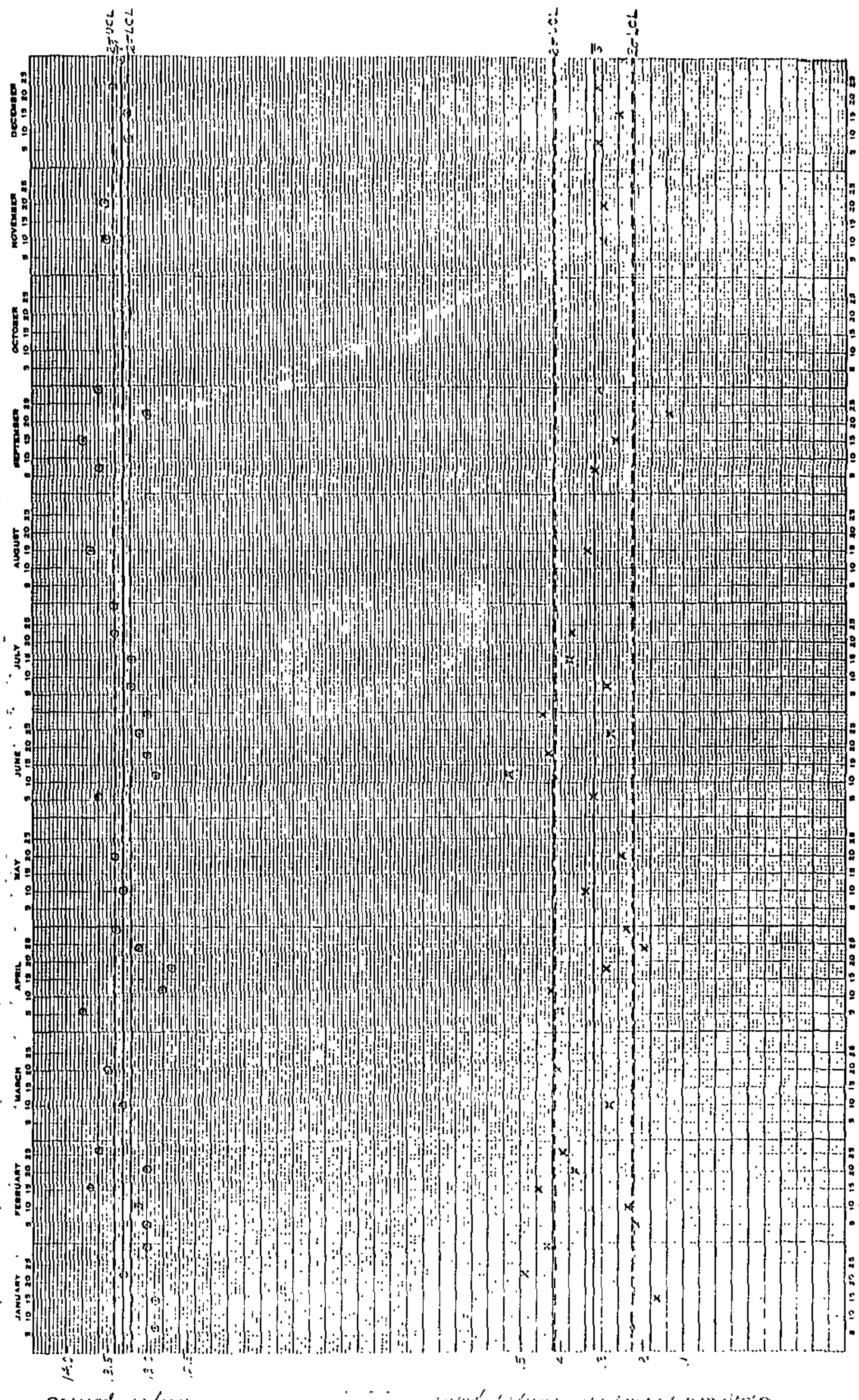

Figure 25

Caliper—Mill H

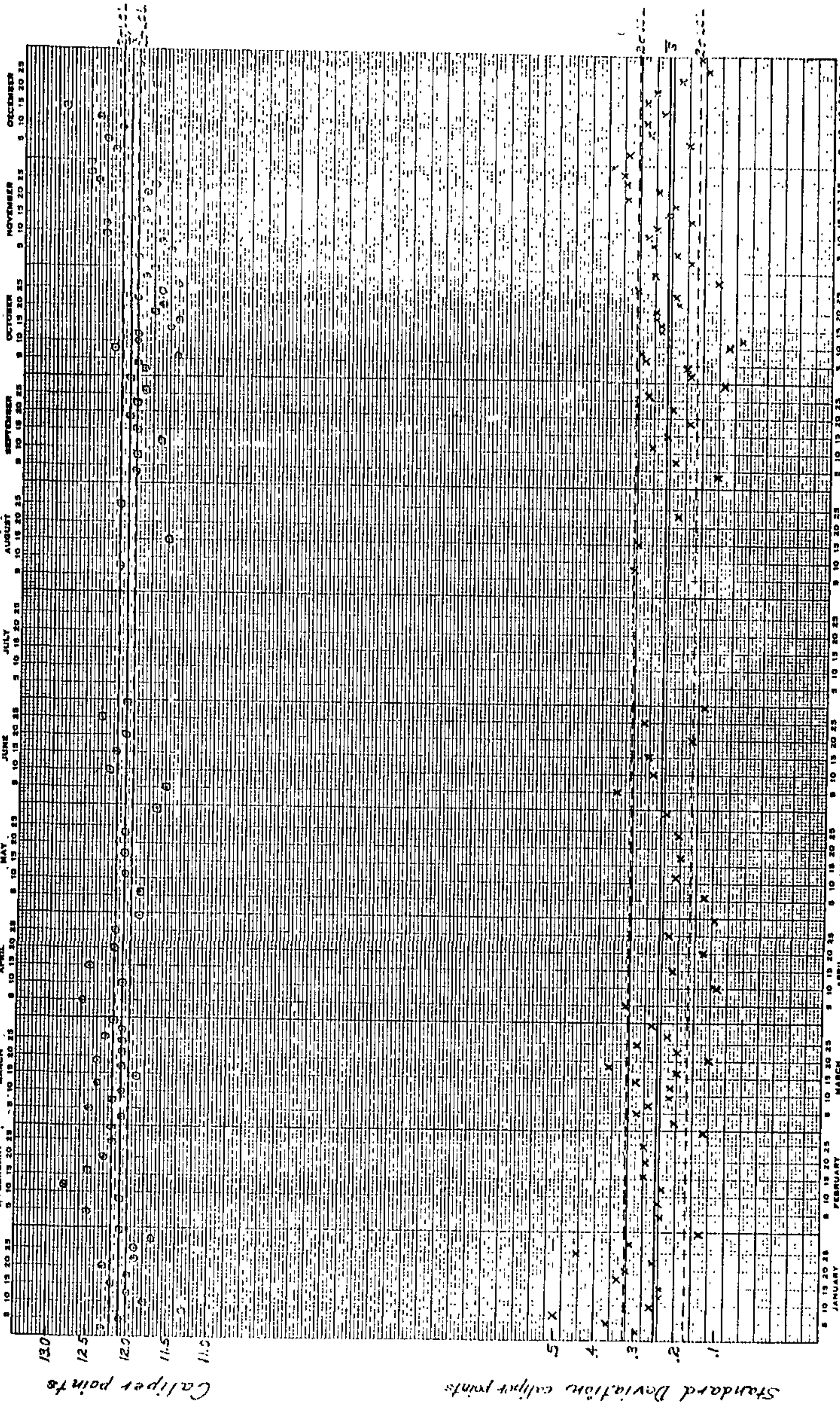

Figure 26

Caliper--Mill I

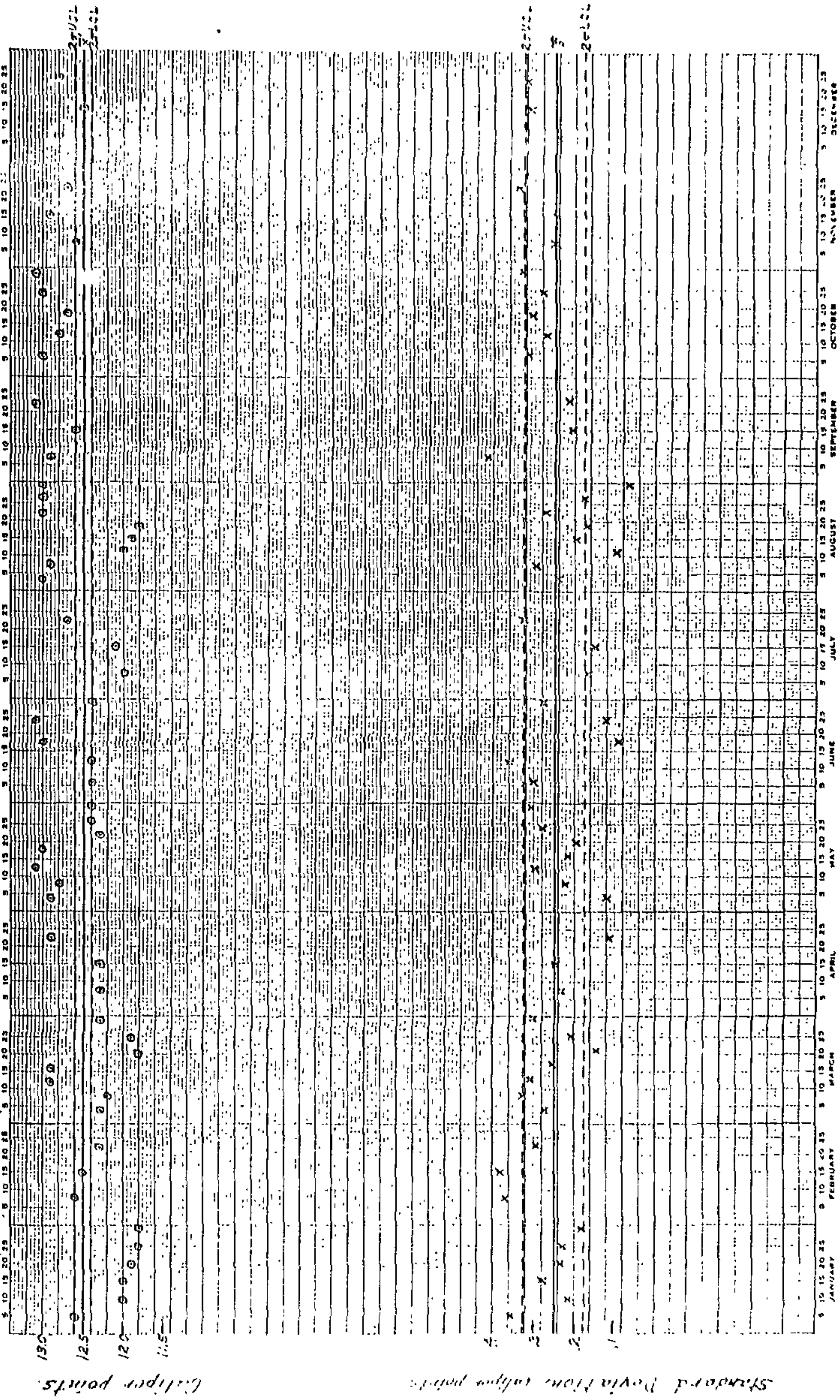

Figure 27

Caliper—Mill J

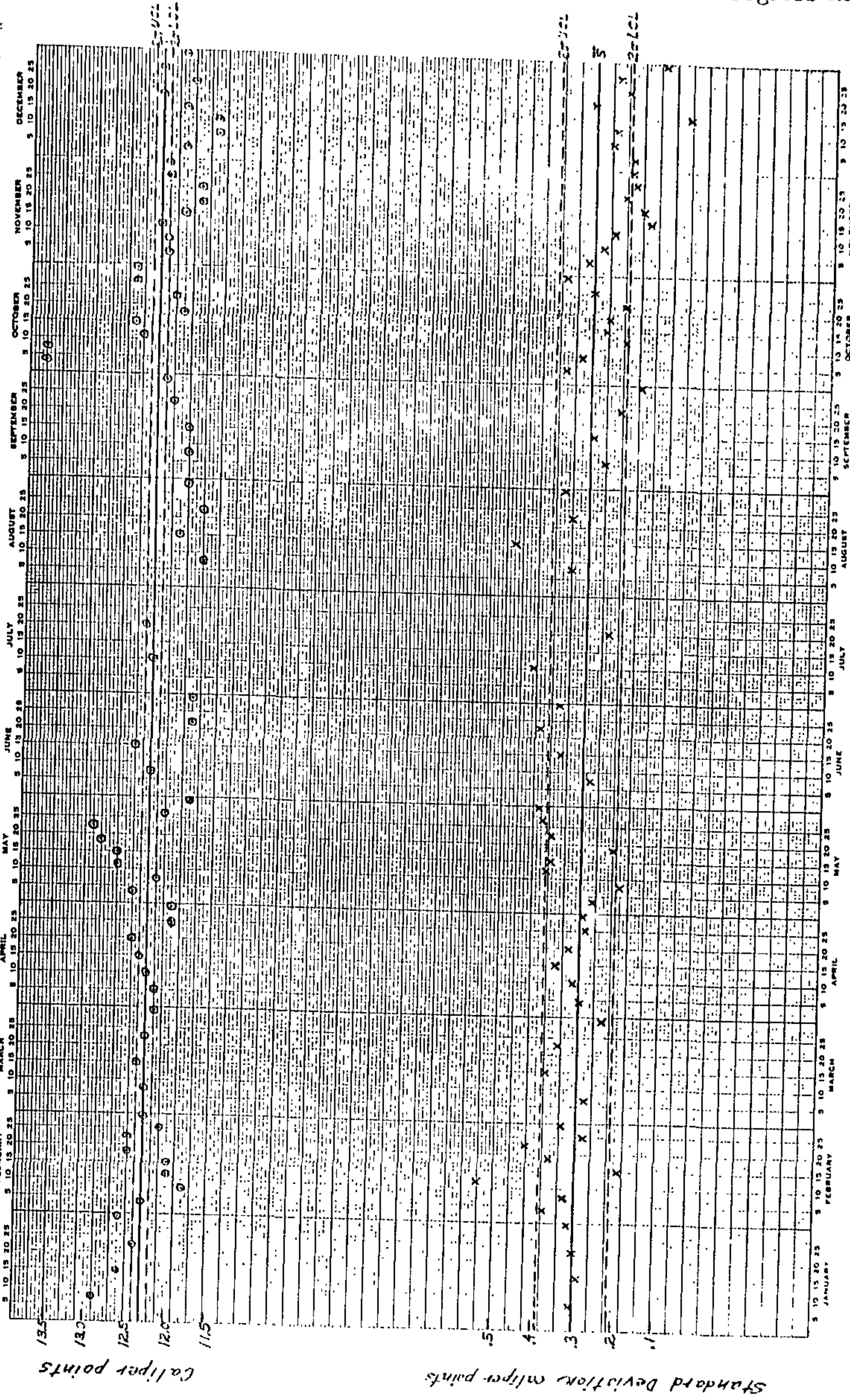

Figure 28

Caliper—Mill K

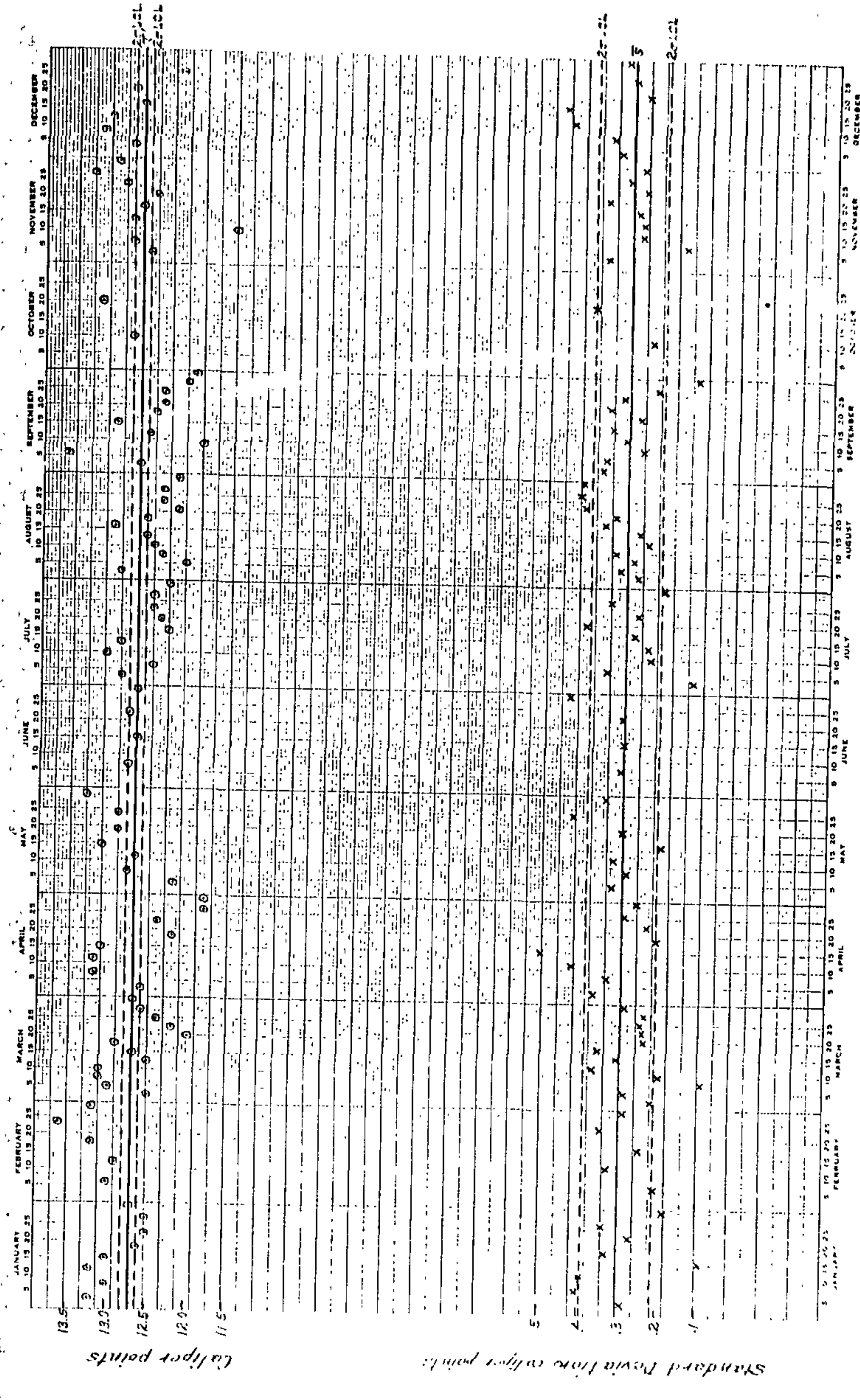

Figure 29

Caliper--Mill L

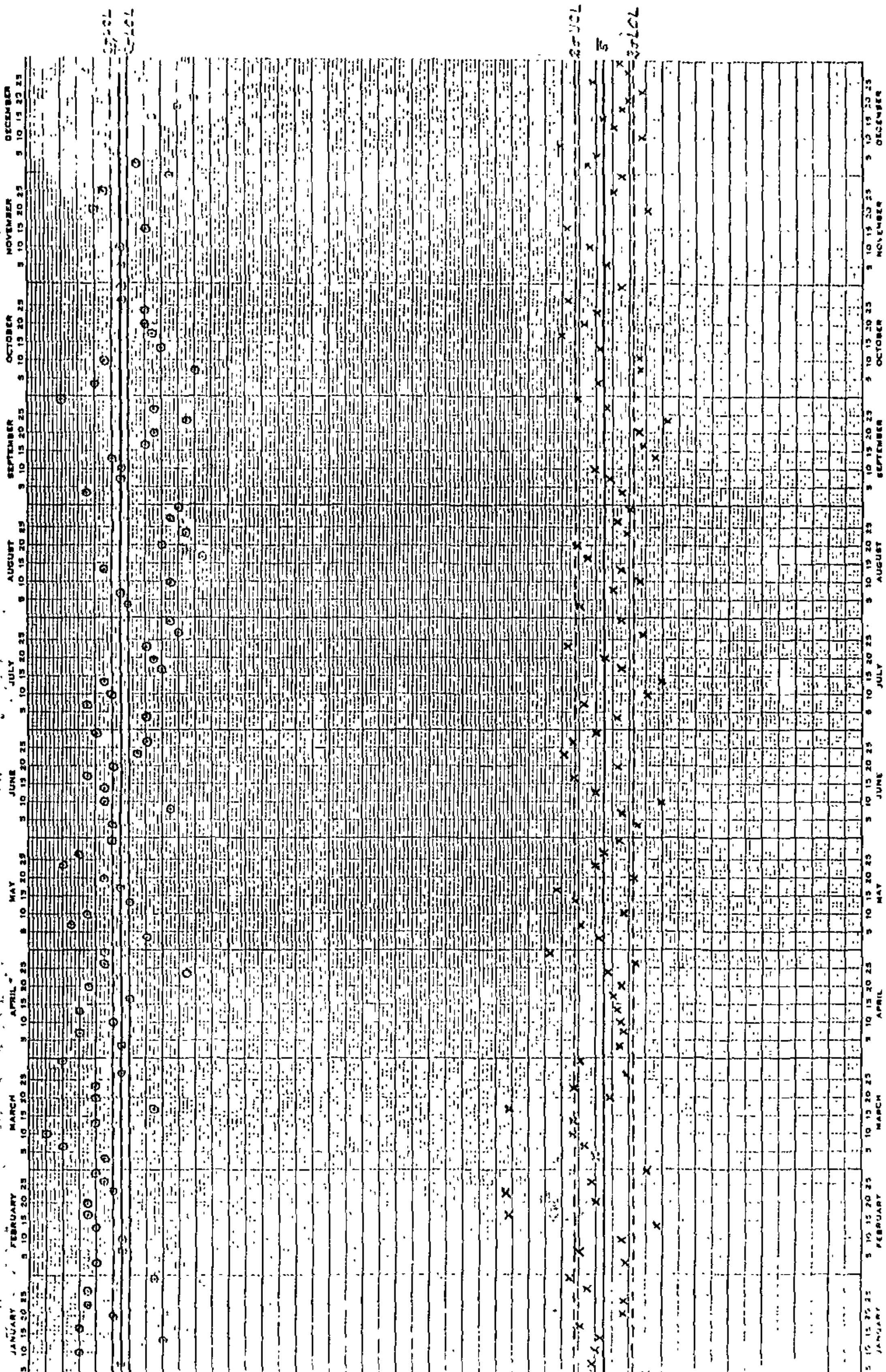

Figure 30

Caliper--Mill M

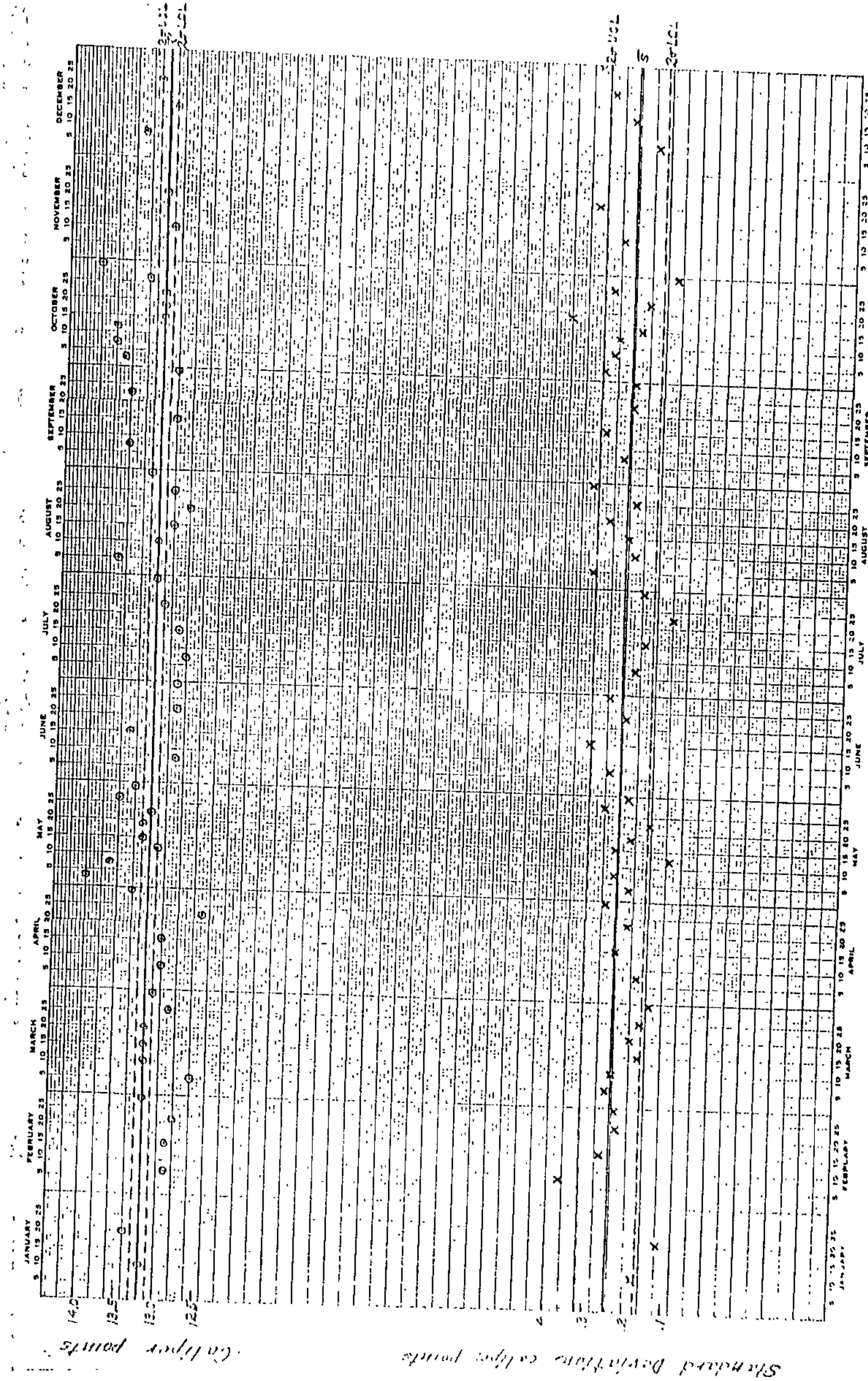

Figure 31

Caliper--Mill N

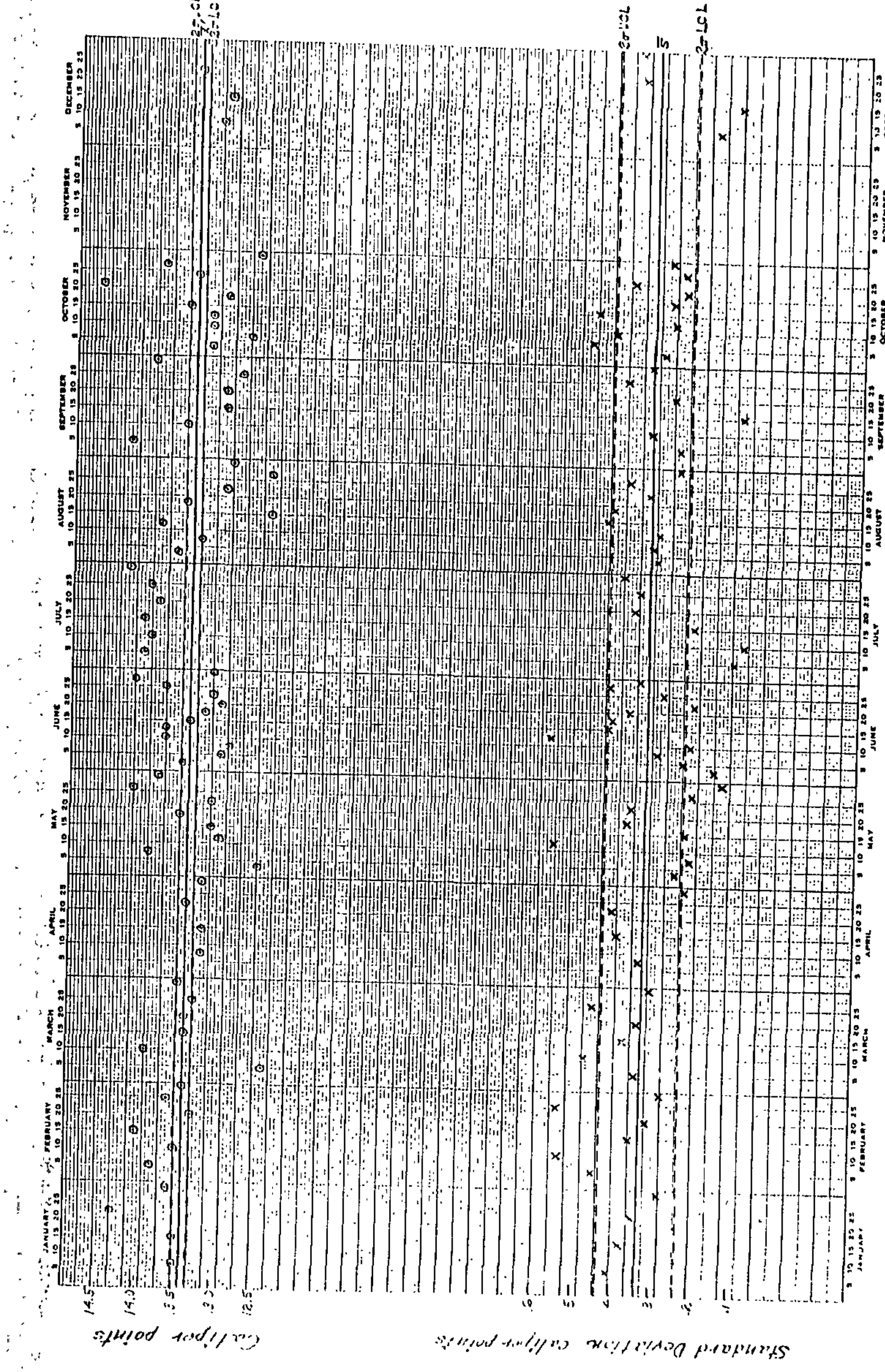

Figure 32

Caliper--Mill 0

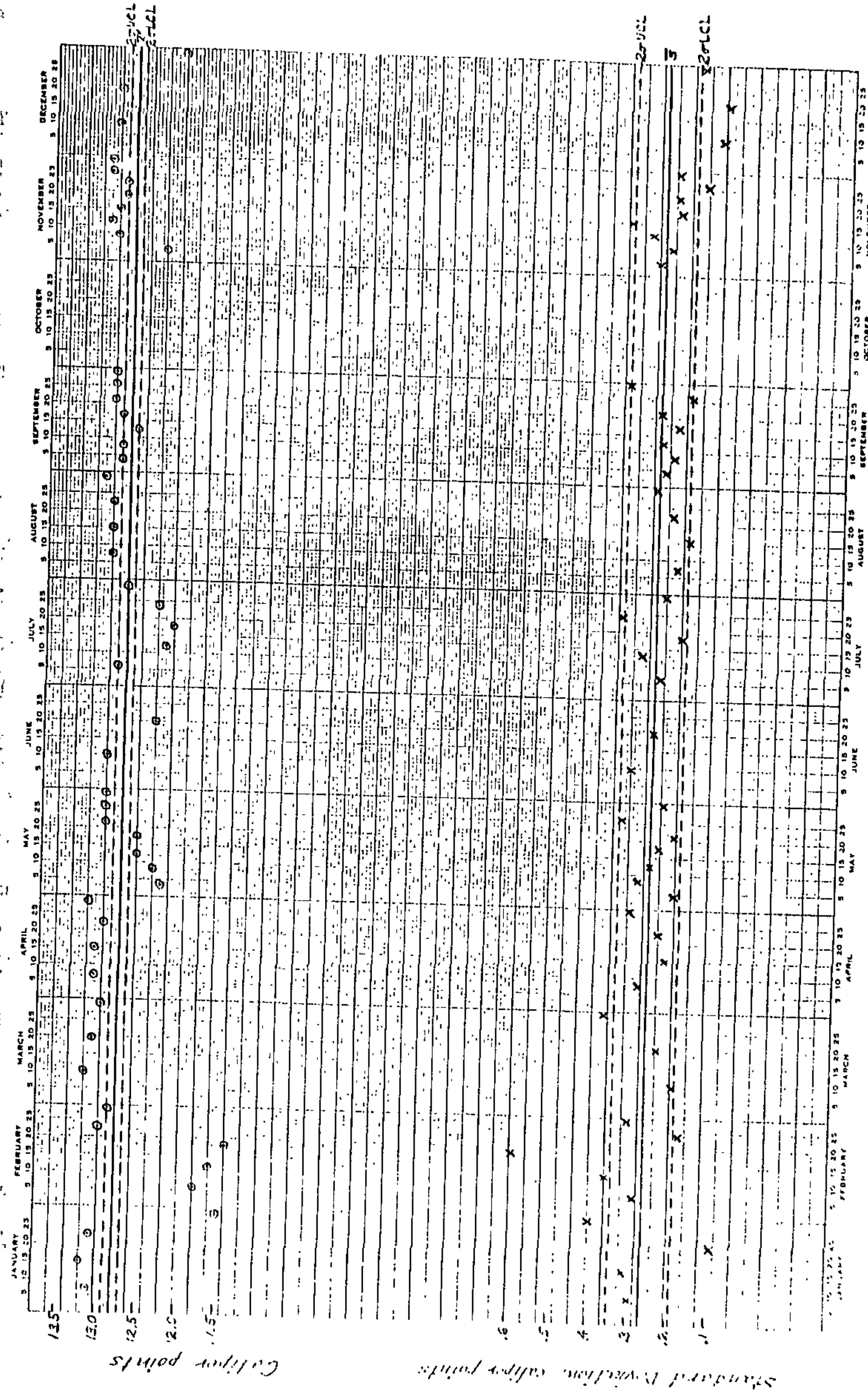

Figure 33

Caliper--Mill P

the result that most of the reel averages fall outside the control limits. The control chart for Mill P may be of particular interest. Examining the chart, it may be noted that the great majority of the reel averages are near 13 points; the over-all average of 12.8 points reflects the presence of a few relatively low caliper samples in February, May and July. Disregarding the low values, if the grand average were placed near 13.0 points, a relatively large number of the remaining averages would fall within the control limits indicating a close approach to statistical control.

Despite the relatively large number of reel averages which fall outside the control limits, it may be noted that the sample standard deviations have a much greater tendency to fall within their control limits. For example, for Mill A about 50 reel averages out of 62 fall outside of the control limits based on within-reel variability. Only five of the sample standard deviations fall outside of their control limits. The same situation holds for most of the other mills. Thus, while the average reel caliper varies considerably, the within-reel or cross-machine variability remains relatively constant. These results are, therefore, in agreement with the weight results discussed previously.

BURSTING STRENGTH

The comparisons of within and between reel variability for bursting strength are summarized in Table V, and the frequency distribution of reel averages are summarized in Table VI. In Table V it may be noted that in terms of per cent 2 standard deviation the within-reel variability ranged from 16.43% for Mill N (Mill Q excluded) to 24.85% for Mill F. The composite

TABLE V

COMPARISON OF THE VARIABILITY WITHIN AND BETWEEN REELS FOR BURSTING STRENGTH BY MILLS

Mill	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	Composite
									Within Reel									
No. of samples	52	38	64	98	41	29	96	38	98	56	68	92	108	56	74	52	6	1076
Grand av., $\bar{\bar{X}}'$	112	111	110	113	110	103	113	114	113	111	109	116	110	112	116	110	114	112
Av. standard deviation, $\bar{s}$	9.6	11.8	13.1	9.5	9.0	12.4	12.4	12.1	10.1	10.3	9.6	10.7	9.3	8.9	11.4	10.2	8.9	10.5
Estimated population standard deviation, σ	9.9	12.2	13.5	9.8	9.3	12.8	12.8	12.5	10.4	10.6	9.9	11.0	9.6	9.2	11.8	10.5	9.2	10.8
Per cent two standard deviation	17.68	21.98	24.55	17.35	16.91	24.85	22.65	21.93	18.41	19.10	18.17	18.97	17.45	16.43	20.34	19.09	16.14	19.29
Two standard error, $2\sigma/\sqrt{N}$	4.042	4.980	5.510	4.000	3.796	5.226	5.226	5.104	4.246	4.328	4.042	4.490	3.920	3.756	4.818	4.286	3.756	4.410
Per cent two std. error	3.60	4.49	5.01	3.54	3.45	5.07	4.62	4.48	3.76	3.90	3.71	3.87	3.56	3.35	4.15	3.90	3.29	3.94
Two S.E. limits about $\bar{\bar{X}}'$	108-116	106-116	104-116	109-117	106-114	98-108	108-118	109-119	109-117	107-115	105-113	112-120	106-114	108-116	111-121	106-114	110-118	108-116
Two S.E. limits about $\bar{s}$	6.7-12.5	8.3-15.3	9.2-17.0	6.7-12.3	6.3-11.7	8.7-16.1	8.7-16.1	8.5-15.7	7.1-13.1	7.2-13.4	6.7-12.5	7.5-13.9	6.5-12.1	6.2-11.6	8.0-14.8	7.2-13.2	6.2-11.6	7.4-13.6
									Between Reel									
Two standard error, $2\sigma_1/\sqrt{N}$	12.6	9.6	8.6	8.4	7.4	8.6	7.6	8.6	9.0	8.6	10.4	9.4	9.2	6.2	9.2	7.8	5.6	8.6
Per cent two standard error	11.25	8.65	7.82	7.43	6.73	8.35	6.73	7.54	7.96	7.75	9.54	8.10	8.36	5.54	7.93	7.09	4.91	7.68

TABLE VI

FREQUENCY DISTRIBUTION OF BURSTING STRENGTH AVERAGES

Bursting Strength, p.s.i.g.	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	Total	Per Cent	Cumulative, Total	Cumulative, %
126												3			1			4	0.4	4	0.4
125												2						2	0.2	6	0.6
124		1							1			3			2			7	0.6	13	1.2
123							1	1				3			4			9	0.8	22	2.0
122	2			2			1	1	1			2			2			11	1.0	33	3.1
121				4				1	2			1			5			13	1.2	46	4.3
120	3						1	3	4			2	1	1	4			19	1.8	65	6.0
119	5			6			1		3		1	8	3		5		1	33	3.1	98	9.1
118	3	1	1	4			9	1	6	5	1	12			6			49	4.6	147	13.7
117	1	2	2	4	3		1	2	5	3	1	7	5		2			38	3.5	135	17.2
116	4	4	4	6	1		7	2	6	1	5	11	4	4	3	4	1	67	6.2	252	23.4
115	4		1	12	1		17	3	8	4	4	6	1	4	5	4		74	6.9	326	30.3
114	3	3	4	10	4		8	3	10	3	8	9	9	6	8	3	1	92	8.6	418	38.8
113	3	3	6	8	3		6	4	6	2	4	5	6	6	8	7		77	7.2	495	46.0
112	4	7	3	12	2		9	6	8	5	3	3	8	6	6	4	2	88	8.2	583	54.2
111	5		4	5	5	1	7	3	12	7	3	1	12	10	2	3	1	81	7.5	664	61.7
110	5	2	12	7	7		7	3	5	2	6	4	13	7	5	1		86	8.0	750	69.7
109	3	4	5	5	2	1	10	2	3	4	4	6	10	3	2	4		68	6.3	818	76.0
108	2	3	3	2	3	3	5	1	5	7	8	1	10	2	4	7		66	5.9	884	82.2
107	4	1	4	4	4	3	3		4	3	1	2	3	3		5		44	4.1	928	86.2
106	1	2	7	3	3	3	1	1	4	2	3		5	2		3		40	3.7	968	90.0
105	1	1		2		1	1		2	4	3		6	2		1		24	2.2	992	92.2
104	3	1	2	1		2		1	3	3	2		5			3		26	2.4	1018	94.6
103		1	3		3	4					3		2			2		18	1.7	1036	96.3
102	2	1		1			1			1	4	1	1			1		13	1.2	1049	97.5
101		1	1			4							1					7	0.7	1056	98.1
100	1		1								2		2					6	0.6	1062	98.7
99																					
98	1					3												4	0.4	1066	99.1
97			1			1					1							3	0.3	1069	99.3
96						3					1							4	0.4	1073	99.7
95	1																	1	0.1	1074	99.8
94	1																	1	0.1	1075	99.9
93																					
92													1					1	0.1	1076	100.0
Total	62	38	64	99	41	29	96	38	98	56	68	92	108	56	74	52	6	1076			

Note: Underlined values are the grand average.

average for all mills was near 19%. It may be of interest to compare the average results obtained for bursting strength with those previously discussed for basis weight and caliper. This comparison is summarized below:

Test	Variability Expressed as Percent Two Standard Error Within Reel	Variability Expressed as Percent Two Standard Error Between Reel	Ratio of Between to Within-Reel Variability
Weight (N=12)	0.82	3.38	4.1
(N=24)*	0.58	2.39	4.1
Caliper (N=24)	0.94	5.86	6.2
Bursting strength (N=24)	3.94	7.68	1.9

* Corrected to N=24 by multiplying N=12 values by $\sqrt{12/24}$. As may be noted above, the within reel variability of the bursting strength test is appreciably greater than that observed for either the caliper or weight tests. (Note: The comparison should strictly be made on the basis of an equal number of tests or perhaps, more preferably in terms of a per cent standard deviation. This consideration is a factor in the weight values and a strictly correct comparison could be obtained by multiplying 0.82 by the square root of 12/24. Similar considerations hold in the case of the between reel values.)

Of greater interest, perhaps, is the ratio of between to within-reel variability which may be looked upon as a crude measure of the ability to control within limits based on the within reel variability. In the case of weight and caliper the ratios are quite high--4.1 and 6.2, respectively. It may be recalled that the control charts for these two tests indicated, in general, that a large proportion of the reel averages fell outside of the within-reel limits. As indicated above, the corresponding ratio for bursting strength is only 1.9 indicating a closer approach to statistical control within limits based on the cross-machine or within-reel variability. It is believed that examination and comparison of the bursting strength, weight, and caliper charts confirms this reasoning.

The control charts for bursting strength are shown in Figures 34 through 49. Mills C, G, and N in Figures 36, 40, and 47 are examples of reasonably good statistical control. While a few more reel averages fall outside of the control limits than would be expected under "ideal" conditions (5%), the excess number is not great.

The chart for Mill D in Figure 37 appears to illustrate a situation where reasonably good control was maintained except for a few periods where accidental or purposeful shifts in average were made. Thus, in the chart, low averages were obtained for a period in late January and early February and again in March. In June a series of samples were received which exhibited relatively high averages. Reasonable control was then maintained throughout the remainder of the year although a slight upward shift in average appeared to occur in November and December, which had the effect of throwing a number of reel averages above the upper control limit.

As mentioned above, under ideal conditions about 5% of the average can be expected to fall outside of the control limits shown on the charts by chance. An excess number of sample averages falling outside of the limits may, therefore, be looked upon as an indication of lack of control. Such a comparison is shown in Table VII. Keeping in mind that situations similar to that discussed for Mill D may be present in a number of cases, it may be observed that

TABLE VII

BURSTING STRENGTH QUALITY CONTROL

Mill	Total Samples	Reel Average Outside of Control Limits Number	Reel Average Outside of Control Limits % *	Reel Standard Deviations Outside of Control Limits Number	Reel Standard Deviations Outside of Control Limits % *
A	62	29	47	5	8.1
B	38	9	24	2	5.3
C	64	9	14	3	4.7
D	98	29	30	10	10.2
E	41	8	20	5	12.2
F	29	6	21	1	3.4
G	96	9	9	9	9.4
H	38	9	24	3	7.9
I	98	35	36	9	9.2
J	56	19	34	3	5.4
K	68	33	49	4	5.9
L	92	29	32	6	6.5
M	108	32	30	11	10.2
N	56	8	14	8	14.3
O	74	20	27	8	8.1
P	52	14	27	9	17.3

* Based on total number of samples as reference.

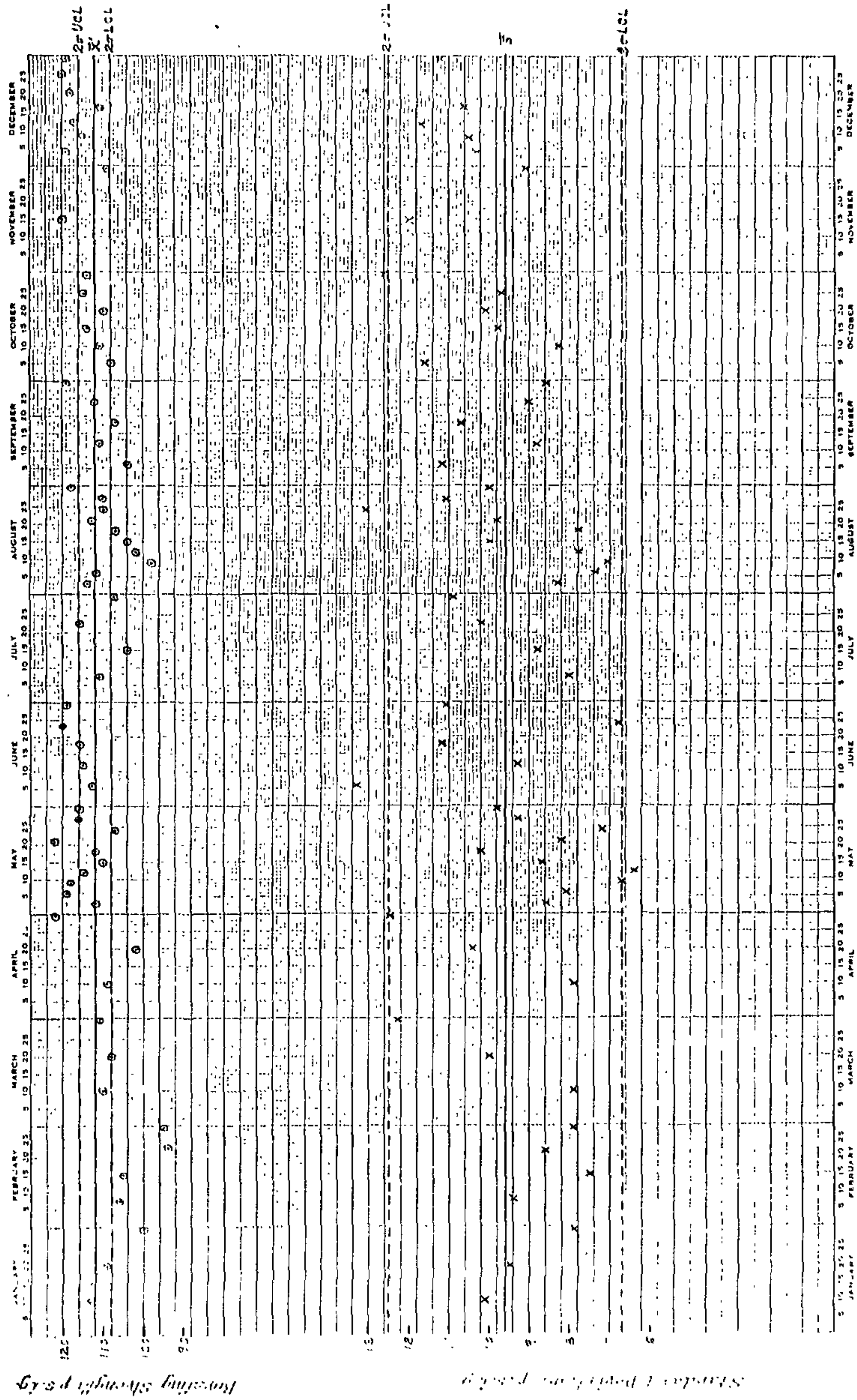

Figure 34

Bursting Strength--Mill A

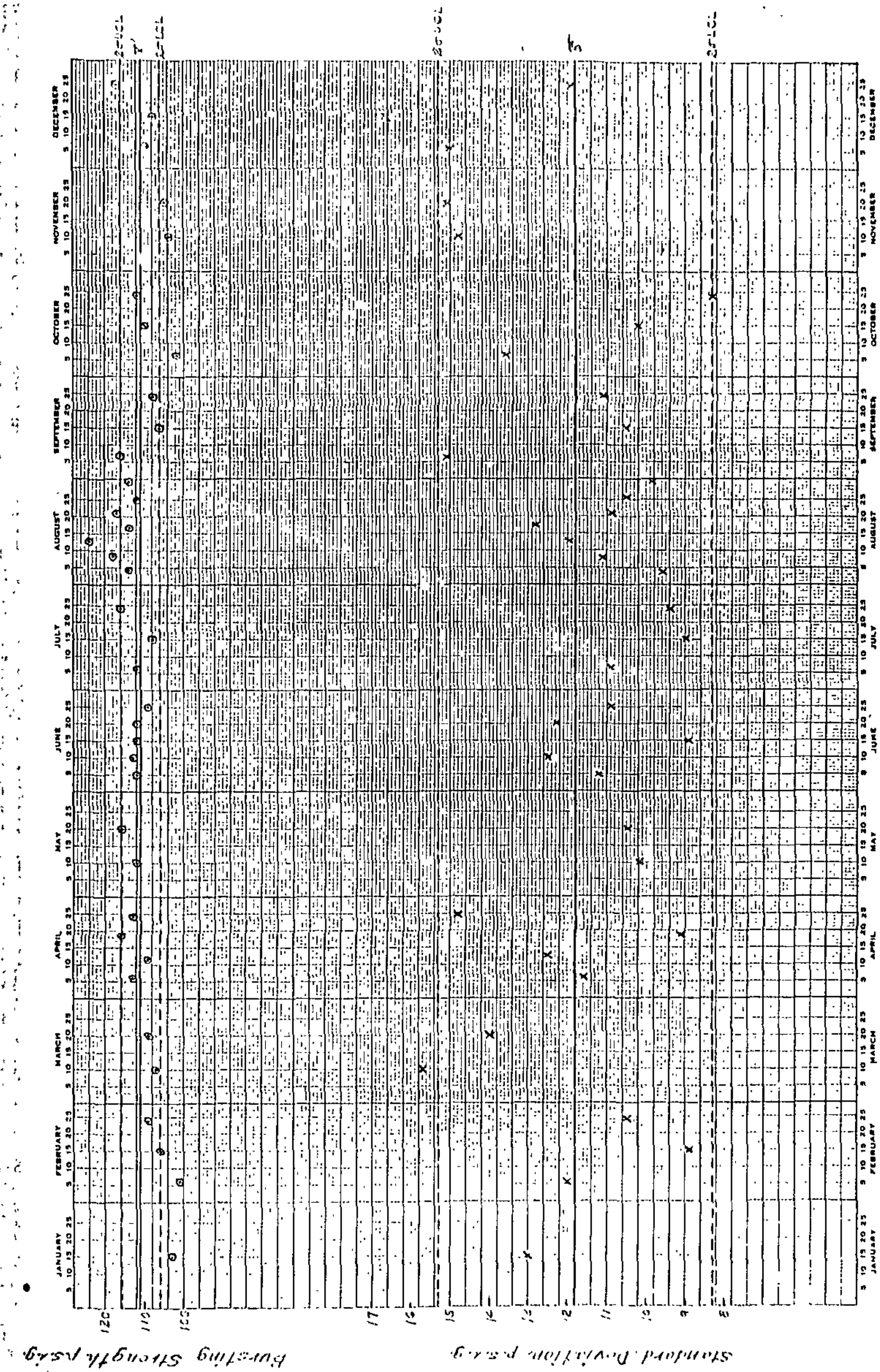

Figure 35
Bursting Strength—Mill B

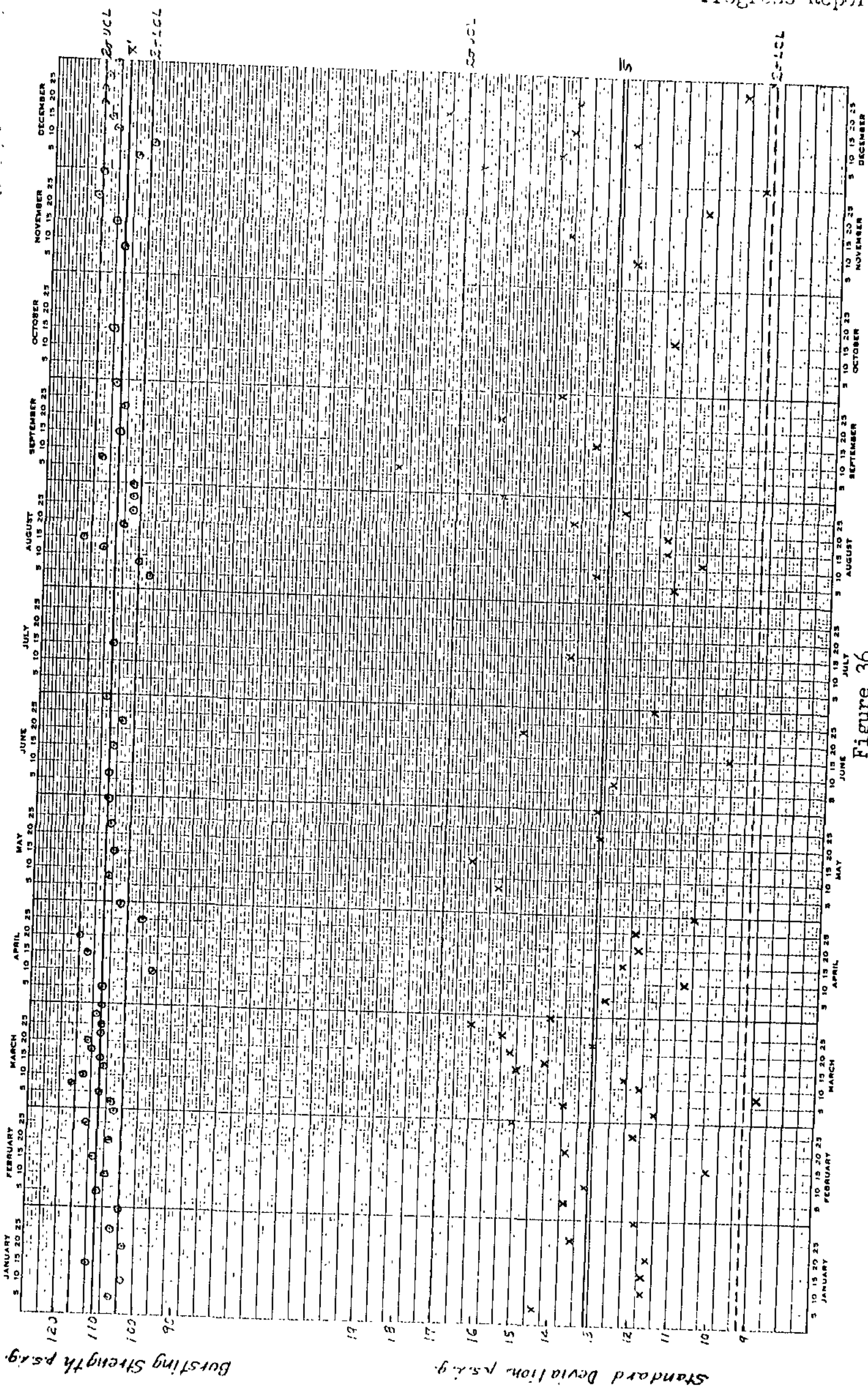

Figure 36

Bursting Strength—Mill C

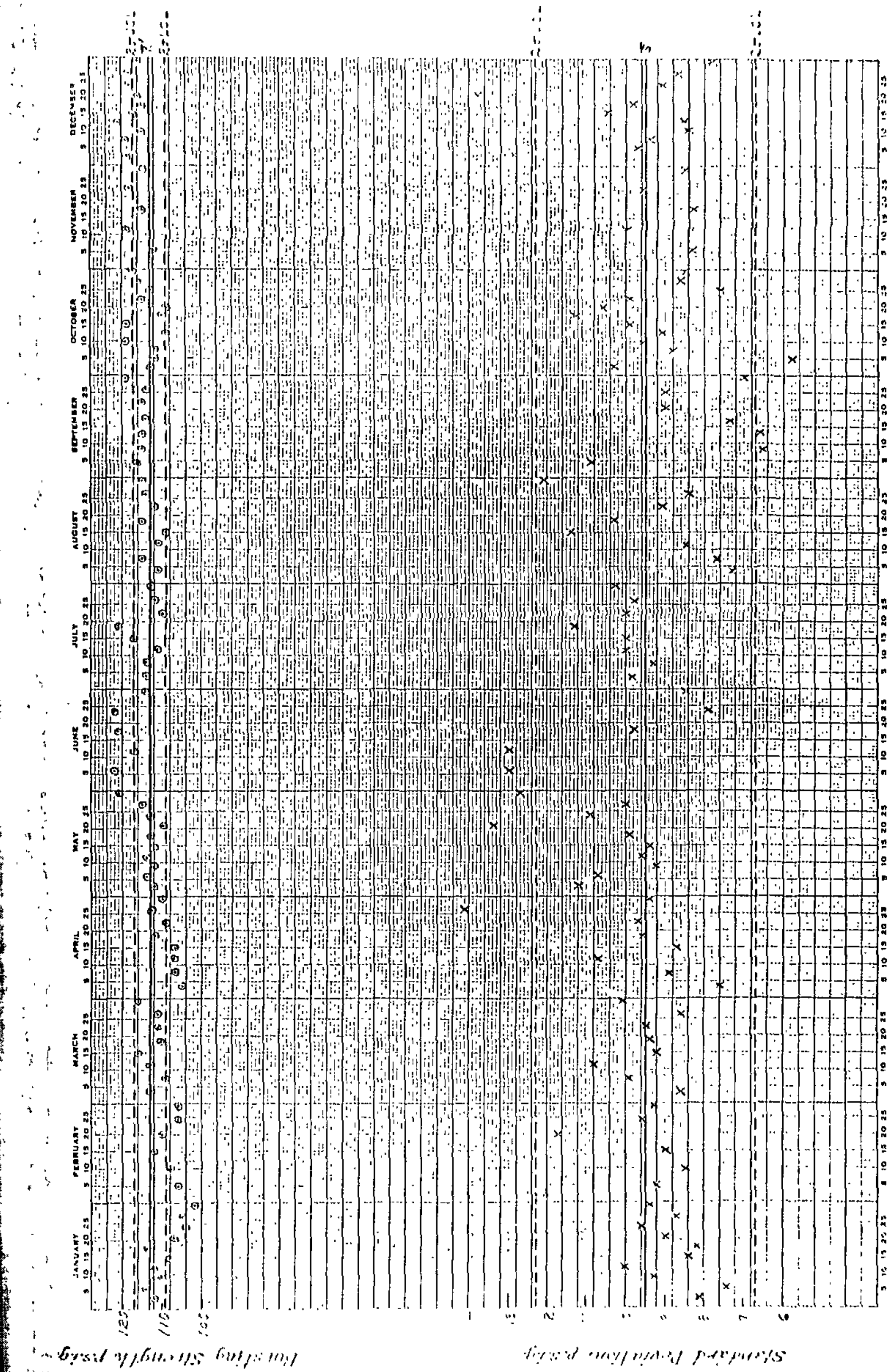

Figure 37

Bursting Strength—Mill D

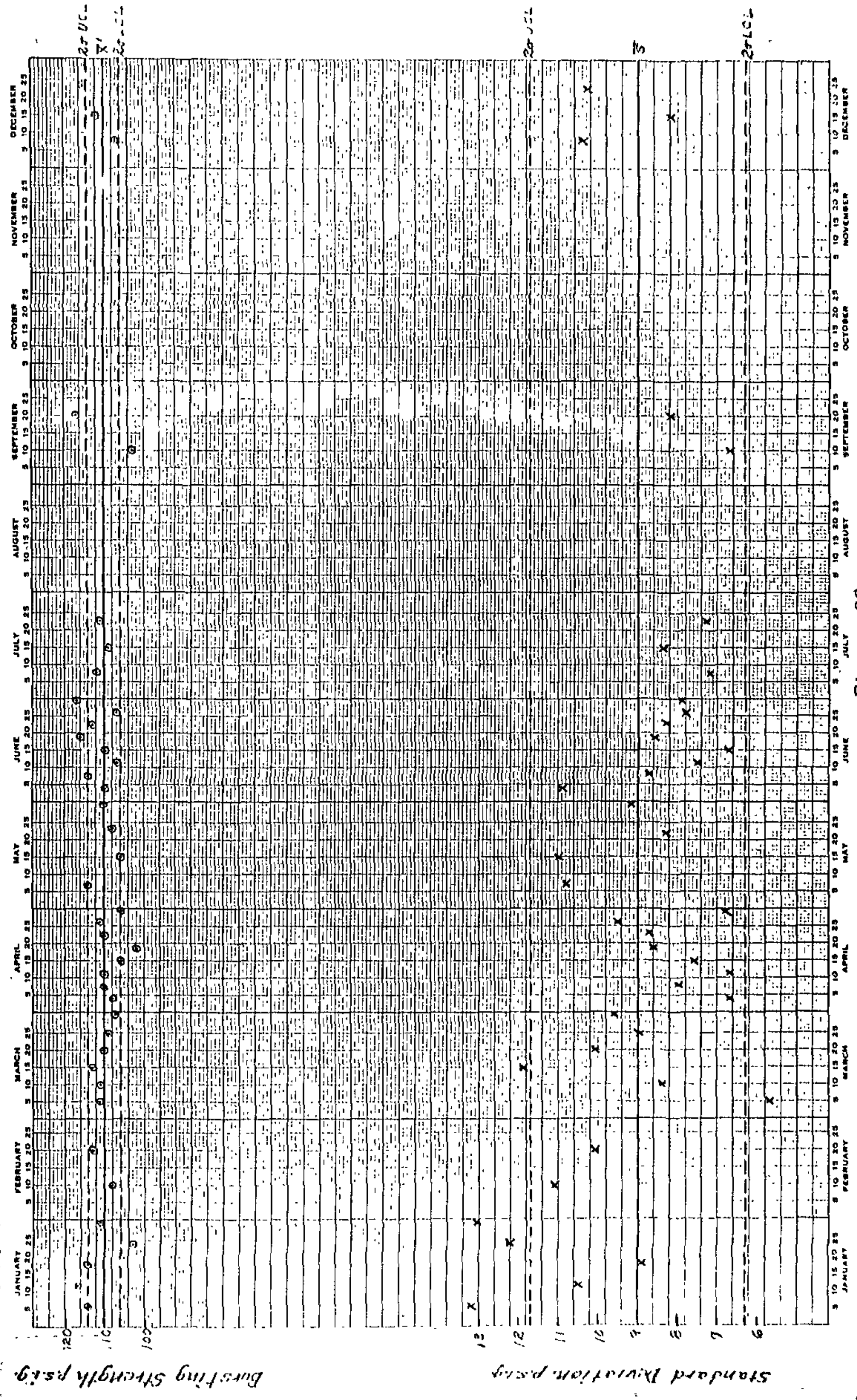

Figure 38

Bursting Strength--Mill E

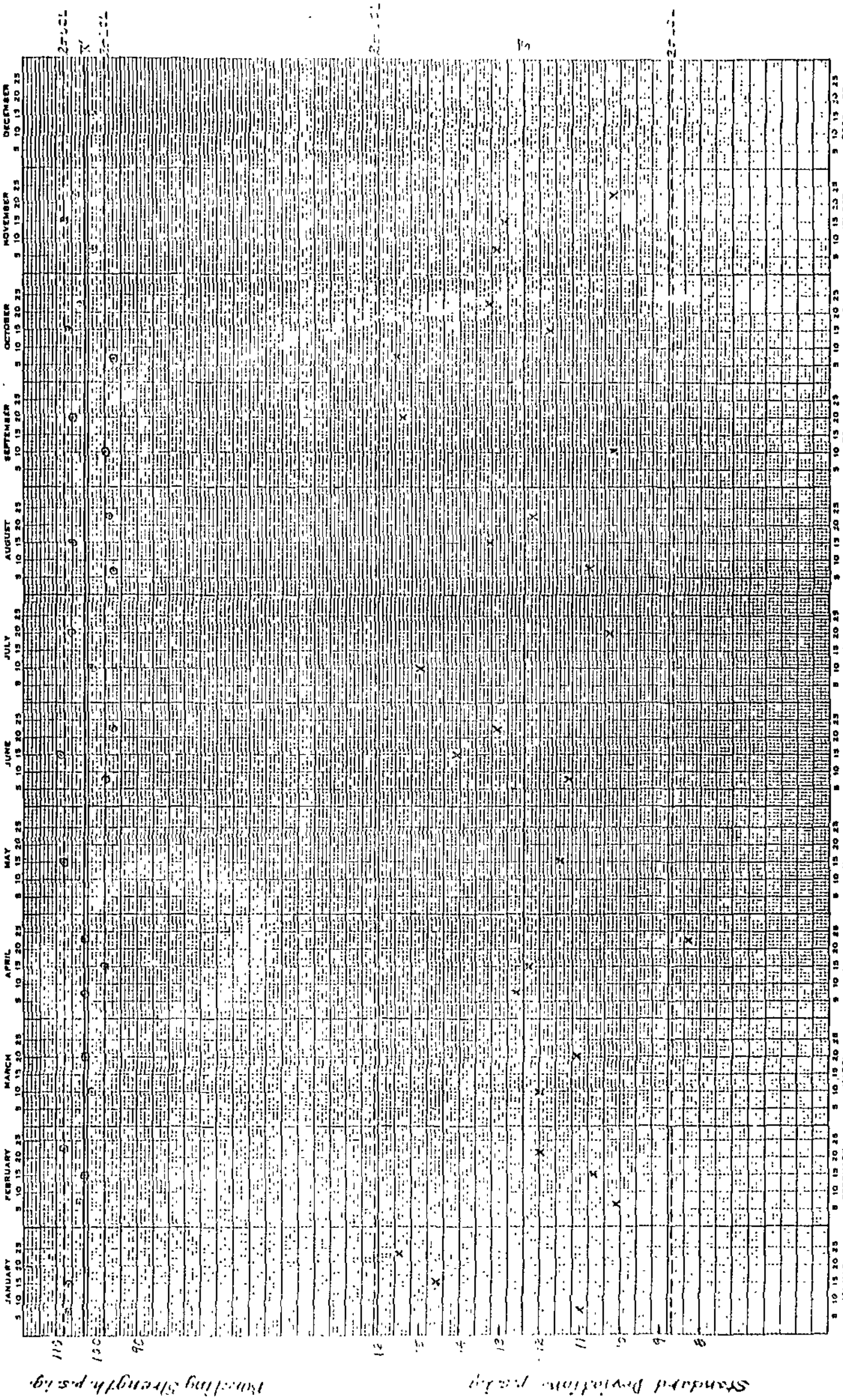

Figure 39

Bursting Strength--Mill F

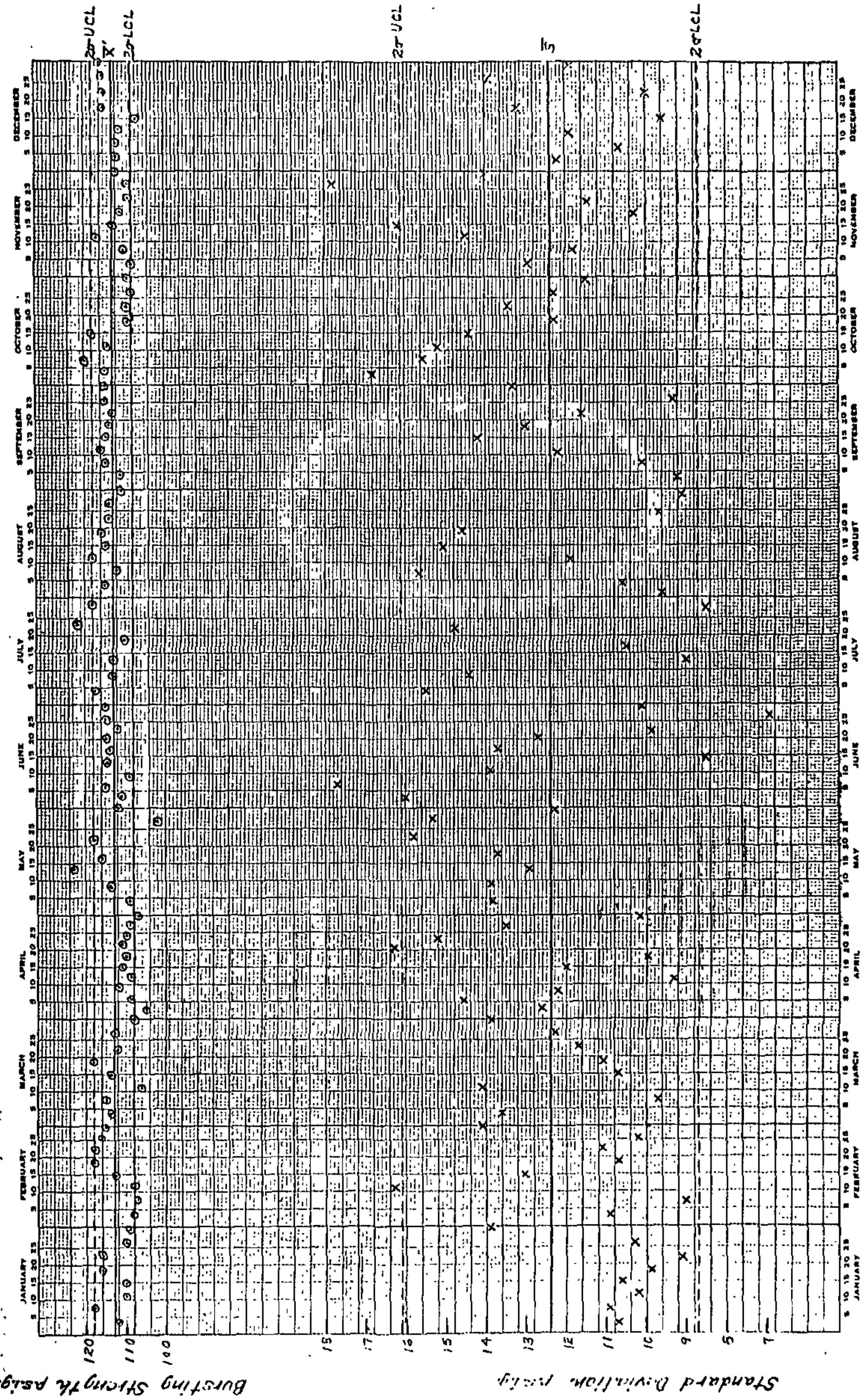

Figure 40
Bursting Strength—Mill G

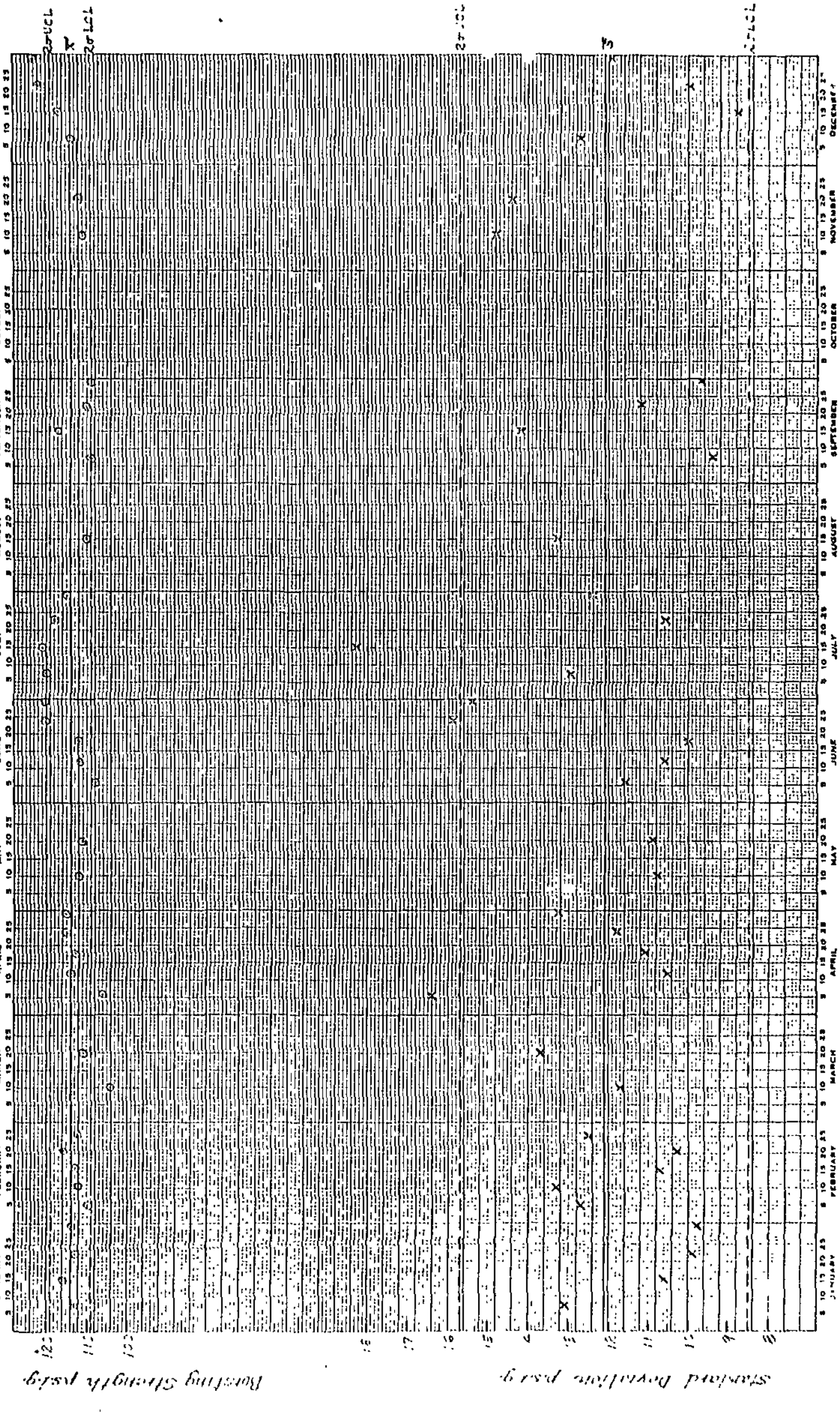

Figure 41
Bursting Strength--Mill H

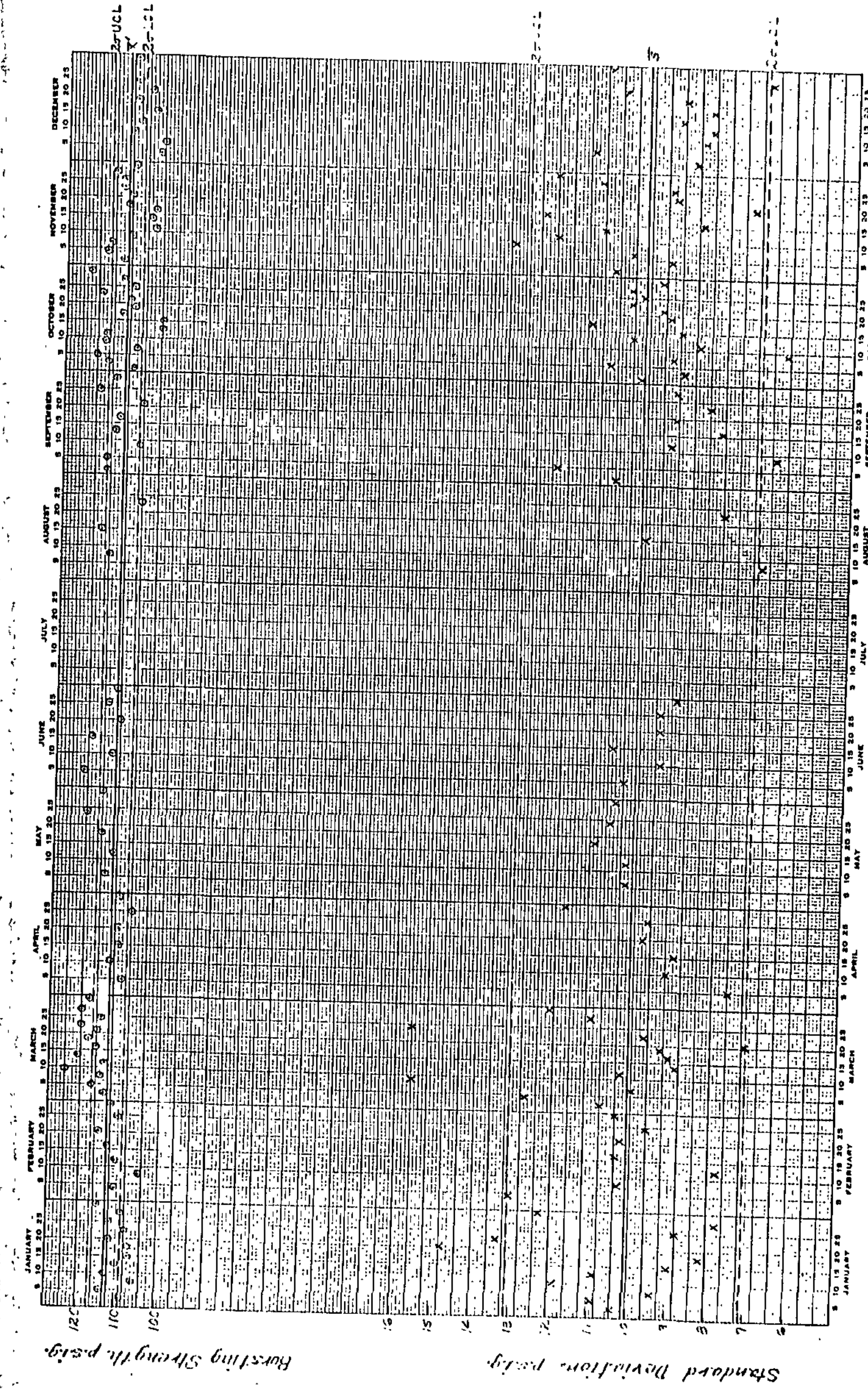

Figure 42
Bursting Strength—Mill I

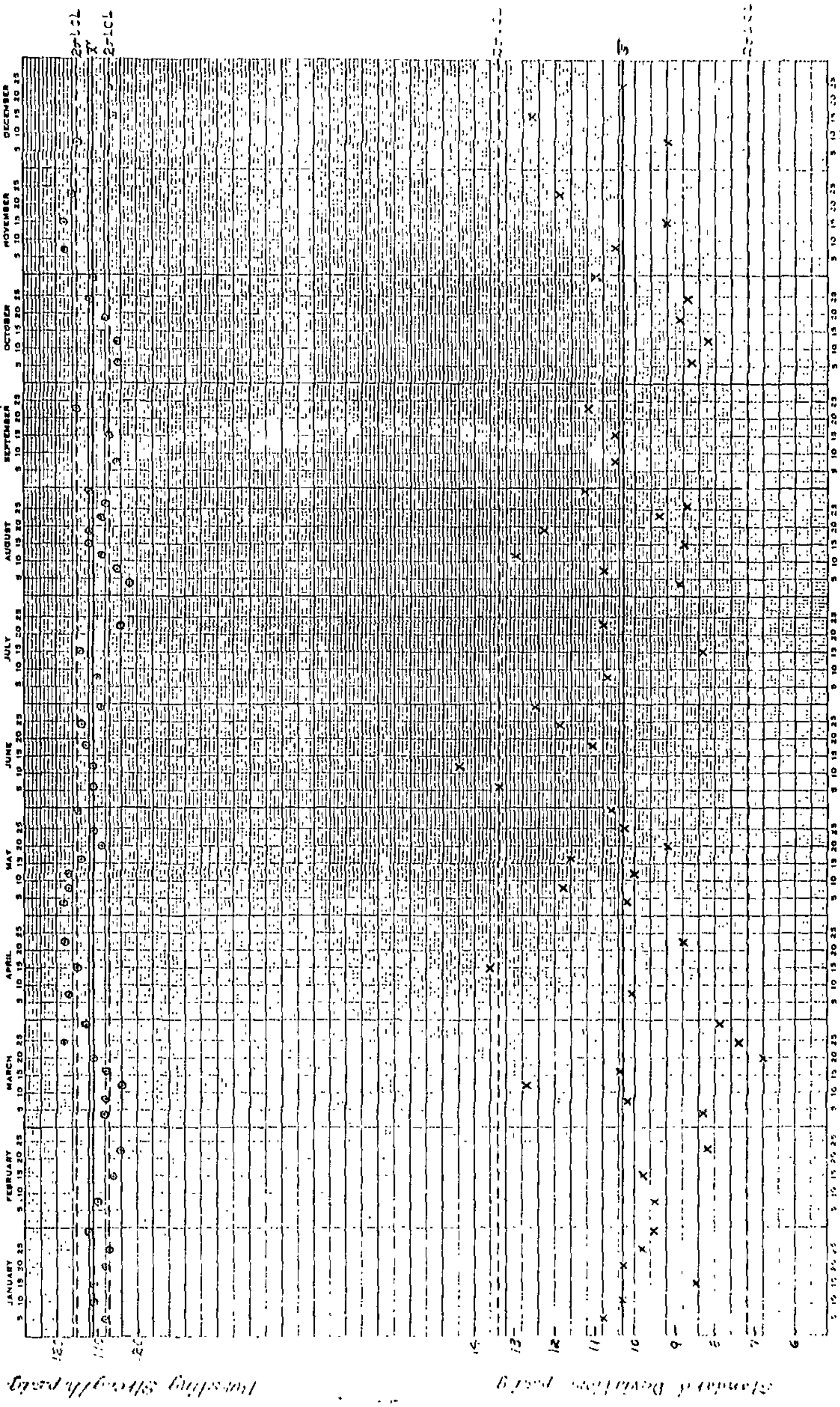

Figure 43
Bursting Strength--Mill J

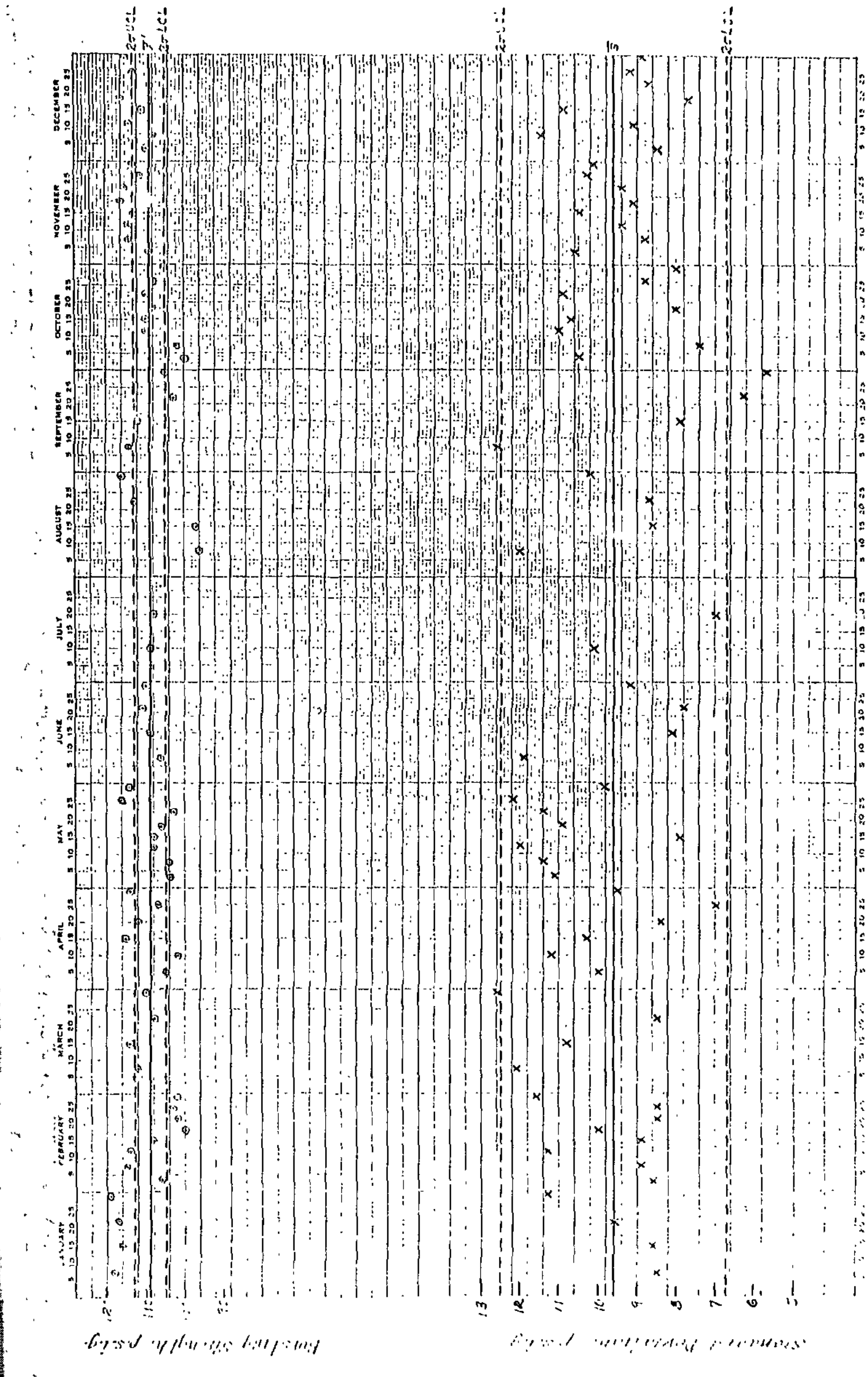

Figure 44

Bursting Strength--Mill K

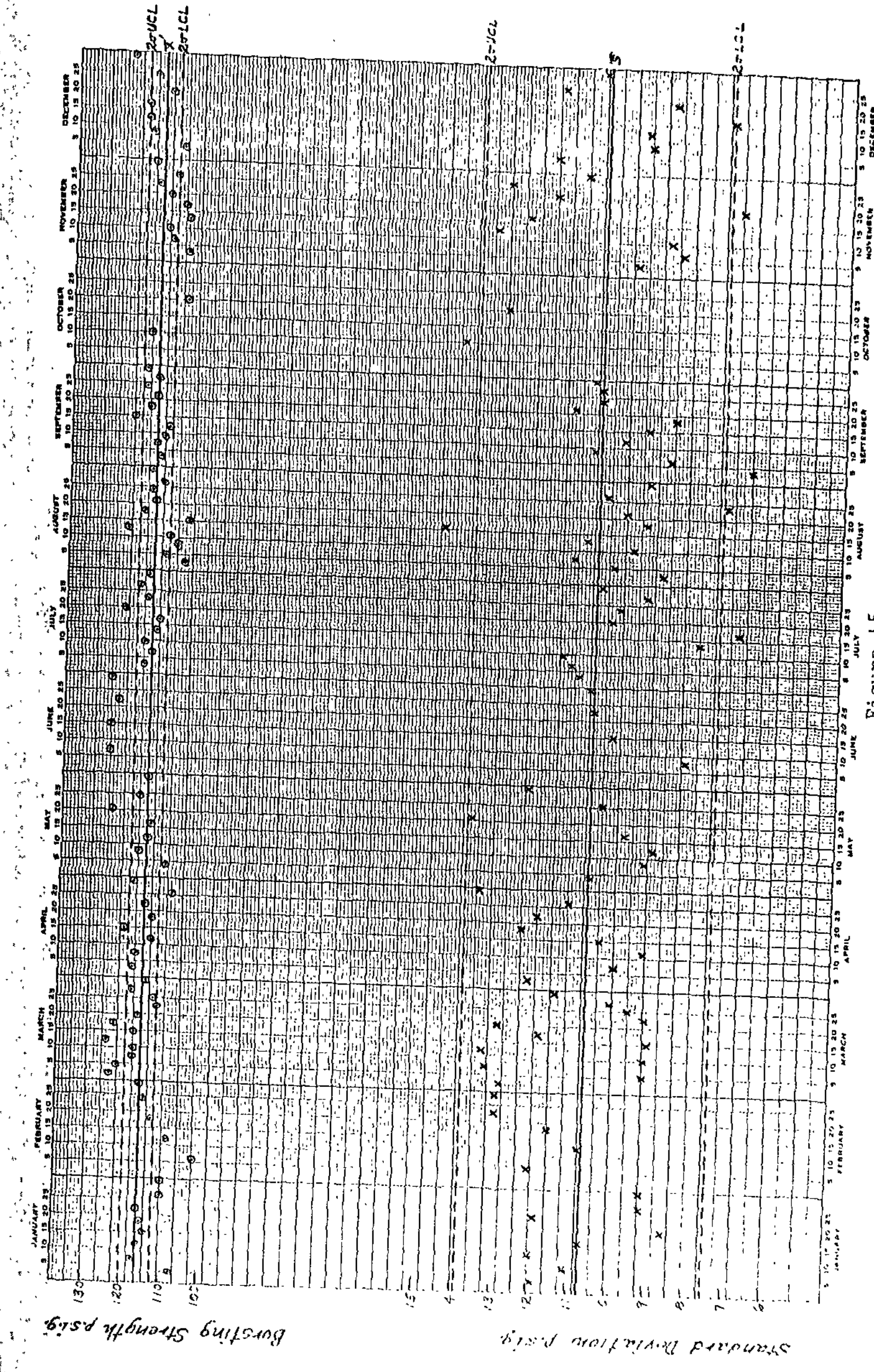

Figure 45

Bursting Strength—Mill L

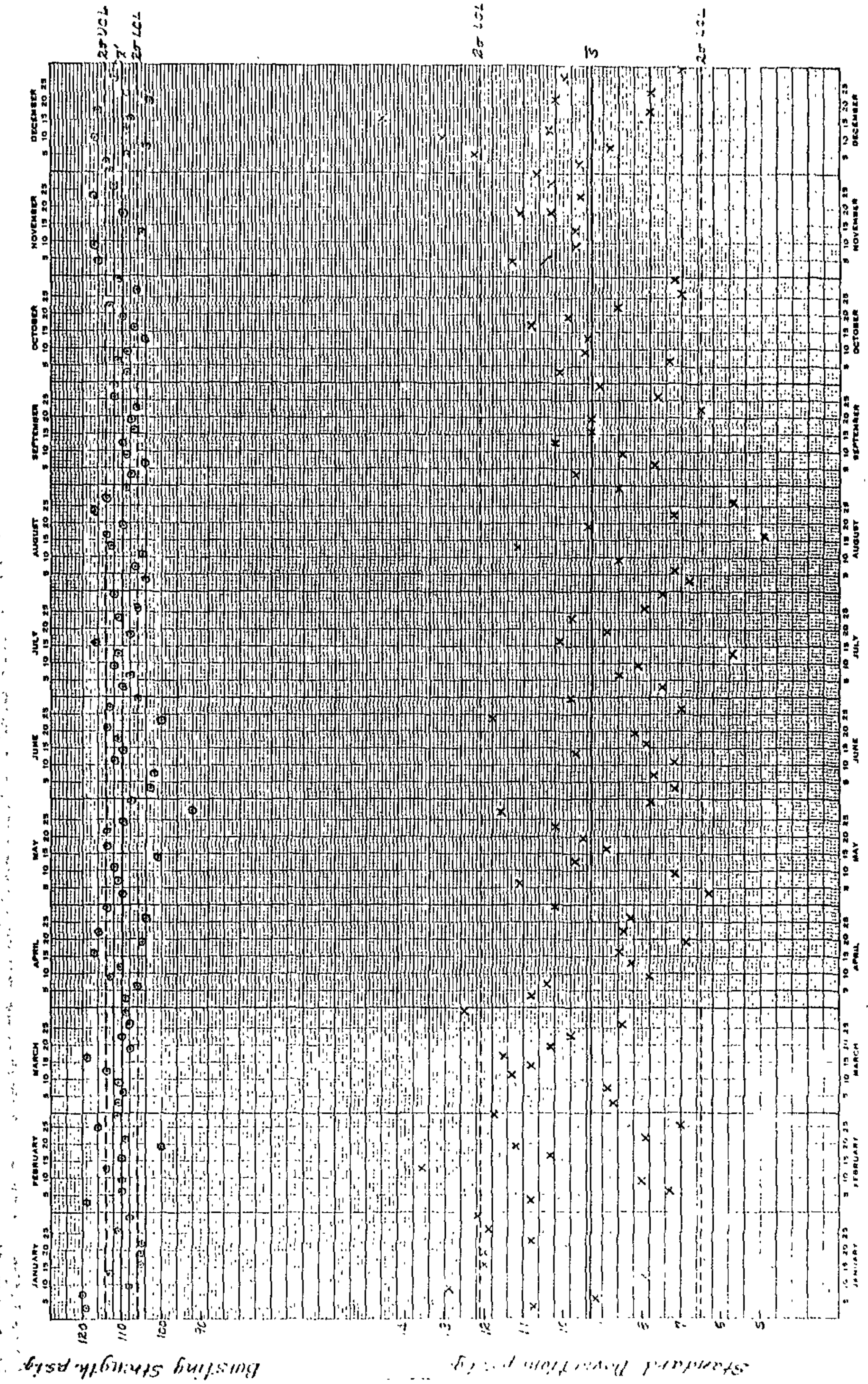

Figure 46

Bursting Strength--Mill M

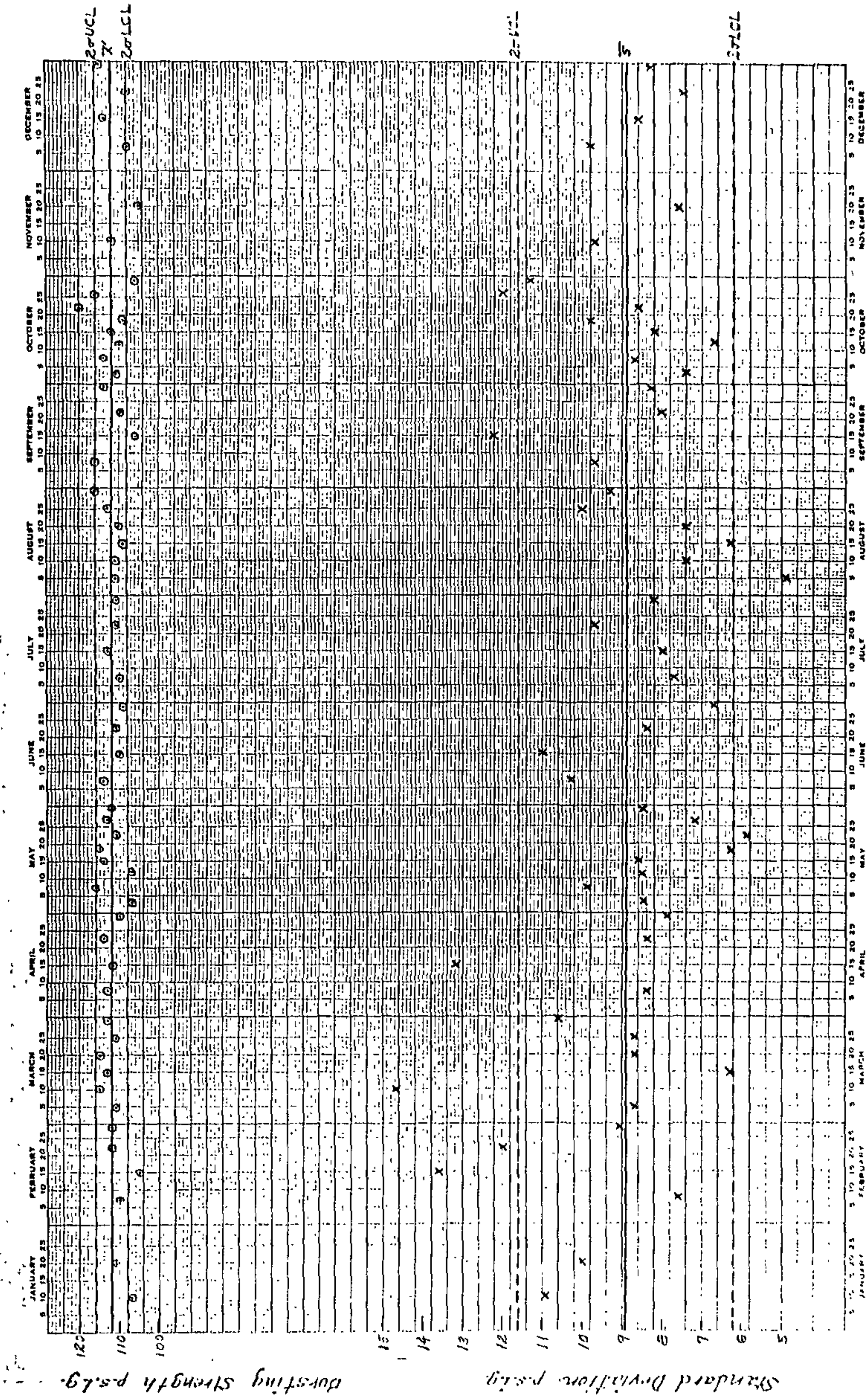

Figure 47

Bursting Strength--Mill N

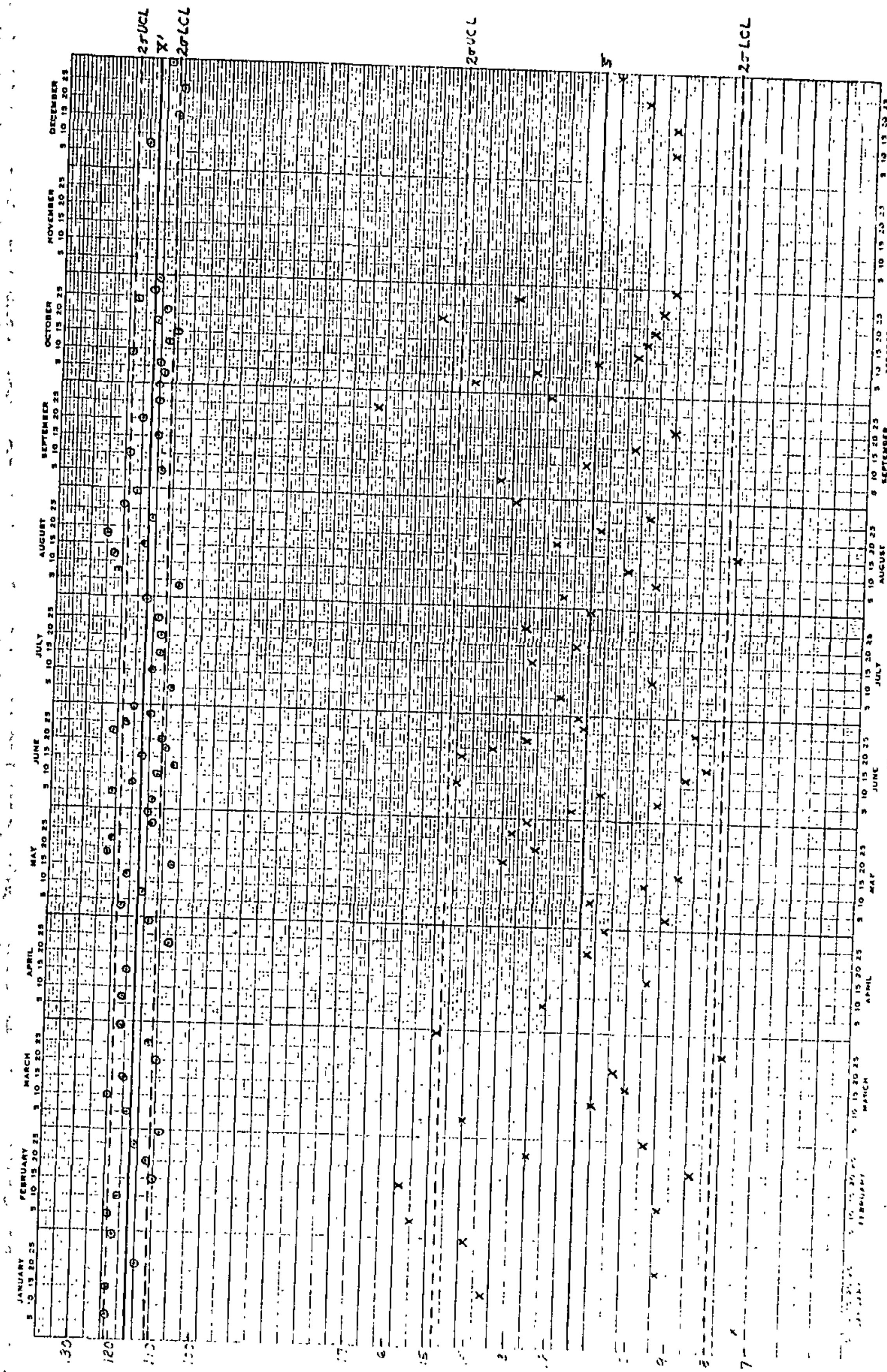

Figure 48

Bursting Strength--Mill O

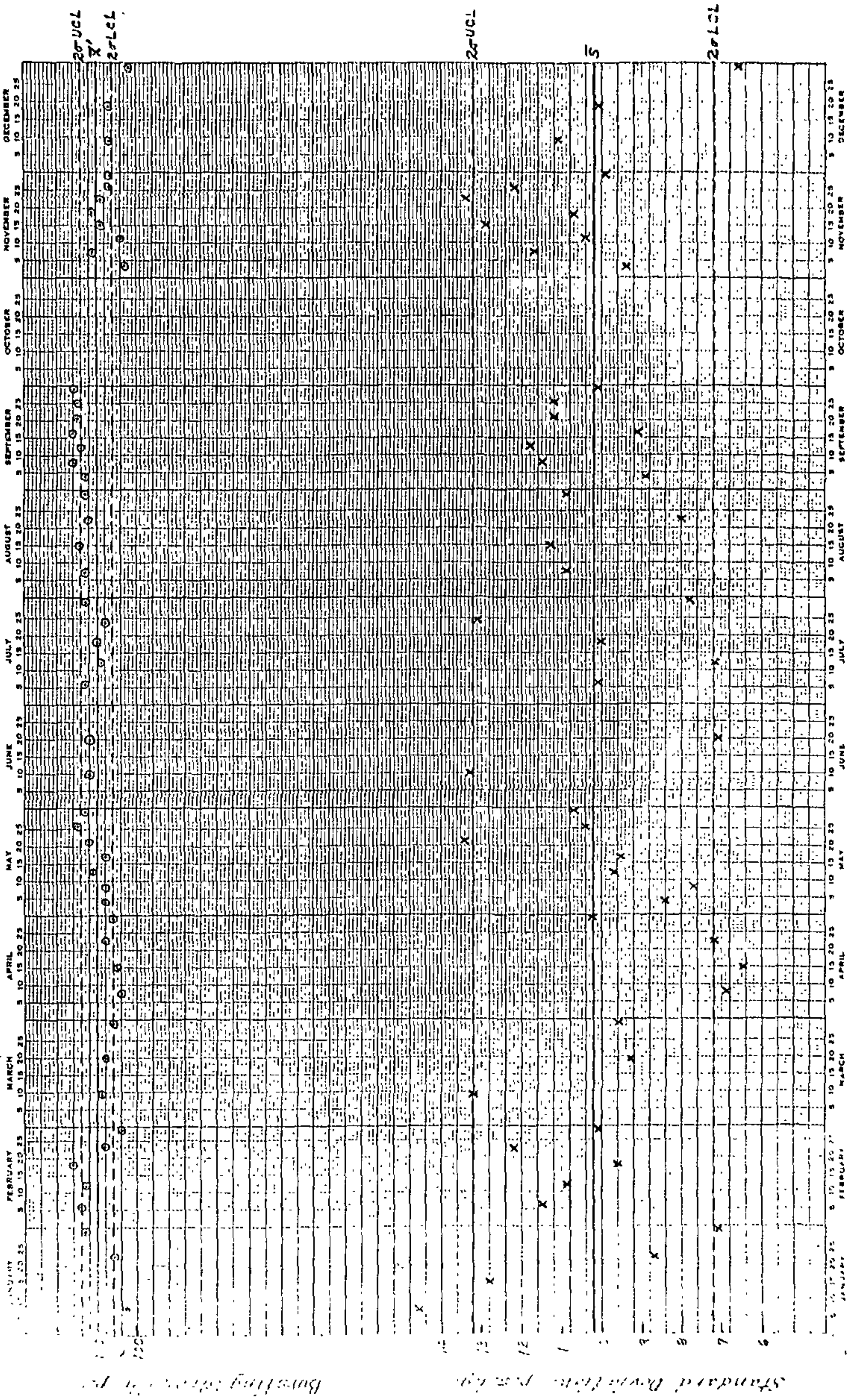

Figure 49
Bursting Strength--Mill P

the reel averages for Mills C, G, and N approach the "ideal" value of 5% reasonably well. Percentages for the remaining mills while quite high in a few cases would be expected to be considerably lower than would be obtained in similar comparisons for either caliper or weight.

With regard to the reel standard deviations in Table VI, it may be observed that an even closer approach to the "ideal" situation is obtained. This fact indicates that the within reel or cross-machine variability is remarkably constant for a given machine which seems reasonable since it is probably a characteristic of the machine which may not be as readily adjusted (in the sense that the weight of the sheet can be adjusted, e.g.). From another standpoint, the close approach to statistical control allows more confidence to be placed in the average sample standard deviation and the population standard deviation derived from it. If sample standard deviations varied widely from reel to reel, product uniformity might appear less satisfactory and would probably be more difficult to improve.

In evaluating the conformance of corrugated boxes to Rule 41, no more than one out of six or four out of twenty-four readings may fall below the minimum specified value. This amounts to approximately 17%. If certain assumptions are made, the estimates of within-reel variability may be used to estimate a minimum reel average to satisfy the above criteria. For example, the bursting strength of corrugated board depends primarily on the bursting strength of the liners; however, the corrugated medium does provide some assistance in the general case. For 200-lb. series corrugated board fabricated with normal quality corrugating mediums, about 20 to 30 points of the bursting strength of the corrugated board may be attributed to the corrugated medium. Taking the lower figure, this suggests that the total liner contribution to the bursting strength of the corrugated board should be at least 180 points or 90 points for each liner. Assuming that no more than 1 out of 6 individual values should fall below 90 p.s.i. g., the minimum reel average which will allow about 17% of the individual readings to fall below 90 p.s.i.g. may be computed.

Such values are shown in Table VIII. It should be emphasized that the results obtained depend primarily on the reasoning described above. Other assumptions regarding the contribution of the liner would necessarily lead to somewhat different results. Referring to Table VIII it may be noted that the estimated minimum reel averages range from about 99 to 103 with a composite average for the group of about 100 p.s.i. g. In terms of the frequency distributions reported in Table VI, only about 1.3% of the reel averages fail to meet this criterion.

Because of the various departures from the "ideal" that have been previously discussed, it was thought desirable to make a cursory test of the above. For this purpose, the samples submitted by Mill F were examined and the number of readings below 90 p.s.i. g. were tabulated as a function of average. A graph of the results is shown in Figure 50. The number of samples averaged in each point is indicated in a parenthesis beside the plotted value. Considering that the above procedure is concerned with extreme values in the distribution curve, the scatter of the points does not appear to be excessive. In addition, reasonable agreement with the theoretical curve was obtained.

TABLE VIII

MINIMUM REEL AVERAGE BY MILL

Mill	Grand Average	Standard Deviation (Within Reel)	Minimum Reel Average *
A	112	9.9	100
B	111	12.2	102
C	110	13.5	103
D	113	9.8	100
E	110	9.3	99
F	103	12.8	102
G	113	12.8	102
H	114	12.5	102
I	113	10.4	100
J	111	10.6	100
K	109	9.9	100
L	116	11.0	101
M	110	9.6	99
N	112	9.2	99
O	116	11.8	101
P	110	10.5	100
Q	114	9.2	99
Composite	112	10.8	100

* Estimated minimum reel average for no more than 4 out of 24 readings below a bursting strength value of 90 p.s.i.g., based on within reel standard deviation for each mill.

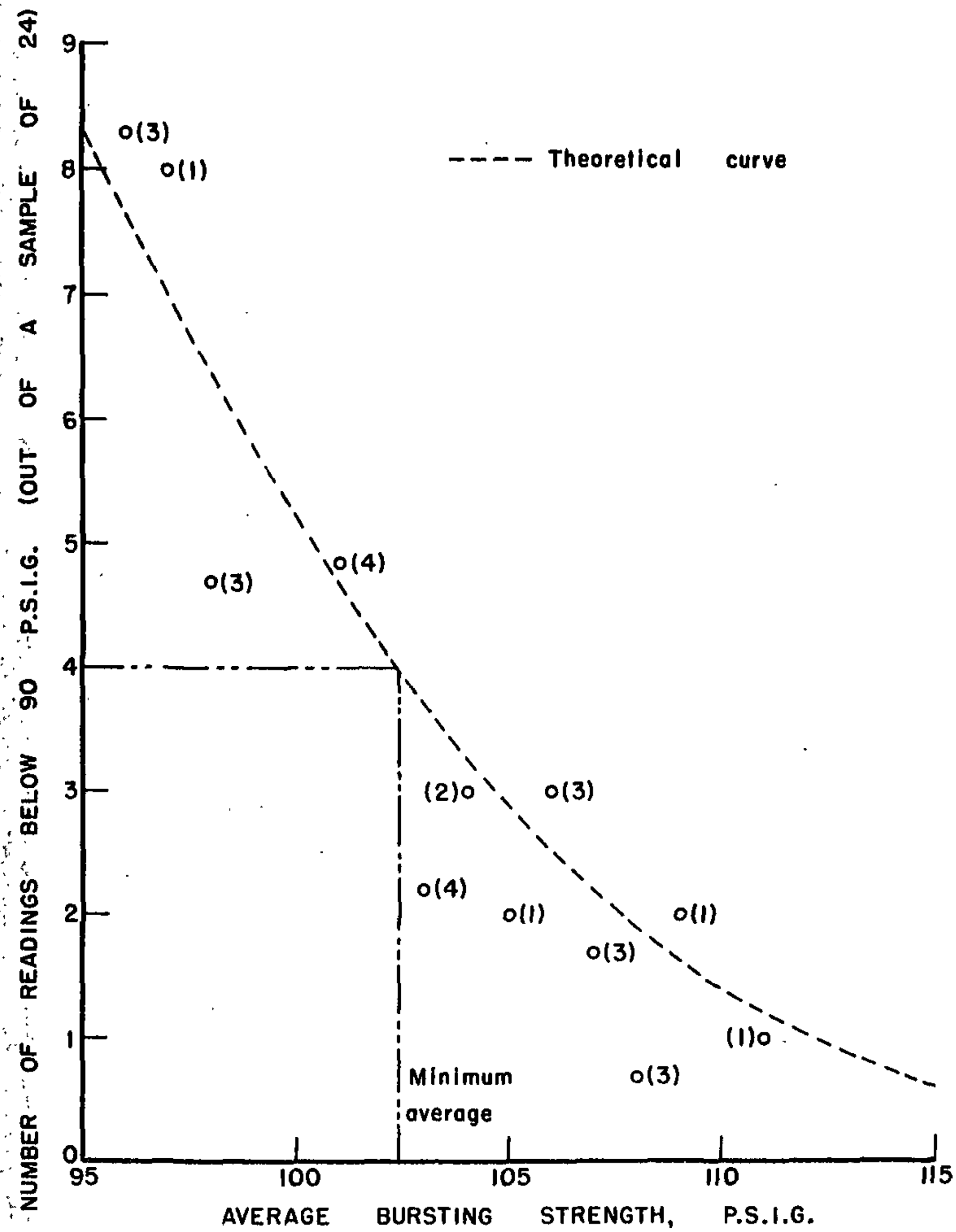

Figure 50. Number of Burst Values Below 90 as a Function of Reel Averages

ELMENDORF TEAR--MACHINE DIRECTION

The comparisons of within and between-reel variability are summarized in Table IX and the frequency distributions of the reel averages are shown for each mill in Table X. With regard to the within-reel variability, a graph of the per cent two standard deviation values vs. mill average is shown in Figure 51. With the exception of one point representing Mill F, the chart indicates that high tear averages tend to be associated with low per cent variabilities.

On the percentage standard deviation basis the within reel variabilities range from 13.63% for Mill N to 20.85% for Mill B. The estimates of between-reel variability in Table IX range from a low of 9.26% for Mill N to a high of 11.87% for Mill F (Mill Q excluded).

Control charts for each mill are shown in Figures 52 through 67. As in the charts previously discussed, the reel standard deviations appear to be in reasonable statistical control for most mills. Reel averages tend to fall outside of their control limits more frequently, however. Definite shifts in average reel test level during the year may be observed for a number of mills. For example, the chart for Mill G in Figure 58 indicates that reels with high tear values were produced in February and a portion of March, while low values were recorded in late October and November. A second example may be found for Mill K. The Mill K results in Figure 62 appear to indicate that reels with somewhat high tear values were produced during most of the year to June. During June, July, August, and September tear values were lower than the over-all average.

TABLE IX

COMPARISON OF THE VARIABILITY WITHIN AND BETWEEN REELS OF MACHINE DIRECTION ELMENDORF TEARING STRENGTH BY MILLS

Mill	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	Composite
Within Reel																		
Number of samples	62	36	64	98	41	29	96	38	98	56	68	92	108	56	74	52	6	1076
Grand av., $\bar{X}'$	338	306	344	317	348	369	314	323	373	316	342	349	351	367	302	296	334	326
Av. standard deviation, $\bar{s}$	29.184	29.856	28.896	25.984	27.120	34.656	28.240	26.720	27.888	29.088	27.392	26.768	26.688	23.408	27.216	26.224	27.776	27.517
Estimated population standard deviation, σ	31.184	31.904	30.880	27.760	28.976	37.024	30.176	28.544	29.792	31.088	29.264	28.608	28.512	25.008	29.072	28.016	29.680	29.402
Per cent two std. deviation	18.45	20.85	17.95	17.51	16.65	20.07	19.22	17.67	15.97	19.68	17.11	16.39	16.25	13.63	19.25	18.93	17.77	18.04
Two standard error, $2\,\sigma/\sqrt{N}$	18.004	18.420	17.328	16.028	16.730	21.376	17.422	16.480	17.200	17.950	16.896	16.518	16.462	14.438	16.784	16.176	17.136	16.976
Per cent two std. error	5.33	6.02	5.18	5.06	4.81	5.79	5.55	5.10	4.61	5.72	4.94	4.73	4.69	3.93	5.56	5.46	5.13	5.21
Two SE limits about $\bar{X}'$	320-356	288-324	326-362	301-333	331-365	348-390	297-331	307-339	356-390	298-334	325-359	332-366	335-366	353-381	285-319	280-312	317-351	309-343
Two SE limits about $\bar{s}$	16.448-41.920	16.832-42.880	16.288-41.280	14.656-37.312	15.296-38.944	19.536-49.776	15.920-40.560	15.072-38.368	15.72[illegible]-40.000	16.400-41.776	15.440-39.344	15.088-38.448	15.056-38.320	13.200-33.616	15.344-39.088	14.784-37.664	15.664-39.888	15.514-39.520
Between Reel																		
Two standard error $2\sigma_{\bar{X}}/\sqrt{N}$	32.8	36.0	36.4	37.4	32.4	43.8	34.8	34.6	39.2	32.2	38.6	38.2	36.4	34.0	31.6	29.2	43.2	36.0
Per cent two standard error	9.70	11.76	10.58	11.80	9.31	11.87	11.08	10.71	10.51	10.19	11.29	10.95	10.37	9.26	10.46	9.86	12.93	10.75

TABLE X

FREQUENCY DISTRIBUTION OF MACHINE DIRECTION ELMENDORF TEARING AVERAGES

Tearing Strength, grams/sheet	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	Total	Per Cent	Cumulative Total	Cumulative %
434-437						1												1	0.1	1	0.1
418-421									3									3	0.3	4	0.4
414-417																					
410-413									1									1	0.1	5	0.5
406-409									4									4	0.4	9	0.8
402-405									4				1	1				6	0.6	15	1.4
398-401						2						2						4	0.4	19	1.8
394-397						1			2				1	5				9	0.8	28	2.6
390-393						2			4		1		1					8	0.7	36	3.4
386-389									8			4		2				14	1.3	50	4.7
382-385	1			1	1	2			1		1		1					8	0.7	58	5.4
378-381					1				8			2	3	4				18	1.7	76	7.1
374-377	1		4		3	2			9		2	2	3	7				33	3.1	109	10.1
370-373	1		4			2		1	8		1	1	5	5				28	2.6	137	12.7
366-369					1	2			6		2	5	7	7				30	2.8	167	15.5
362-365			6		2	3		1	10		3	2	6	4				37	3.4	204	19.0
358-361	3		2		1	3			8		1	7	14	4				43	4.0	247	23.0
354-357	4		1	1	6	3		1	9	1	9	8	9	4				56	5.2	303	28.2
350-353	3		2	1	3	2	1		5		2	9	10	5				43	4.0	346	32.2
346-349	4		10	3	3	2	3	2	1	1	10	8	8	3				58	5.4	404	37.5
342-345	12	2	5	4	5		1		4	2	4	9	9				1	58	5.4	462	42.9
338-341	2	1	5	2	6		5	1	1	1	6	6	6	2				44	4.1	506	47.0
334-337	6		4	4	5	1	7	1		2	1	6	5		1		2	45	4.2	551	51.2
330-333	9		6	7			5	1	1	8	5	5	6	2	1		2	58	5.4	609	56.6
326-329	4	1	3	6		1	4	4	1	1	3	6	3	1	1	1		40	3.7	649	60.3
322-325	4	2	5	6	1		7	6		3	8	5	6		3	2	1	59	5.5	708	65.8
318-321	2	5	5	6	2		6	5		5	3	2	2		7			50	4.7	758	70.4
314-317		3		10			9	2		4	1	2			7	2		40	3.7	798	74.2
310-313	3	3	1	11			4	5		8	1				7	4		47	4.4	845	78.5
306-309	2	4		8	1		13	5		5	1				8	3		50	4.7	895	83.2
302-305		1		5			7	2		4	1	1			6	7		34	3.2	929	86.3
298-301		5		9			8	1		5	2		1		5	5		41	3.8	970	90.1
294-297	1	2	1	4			5						1		6	6		26	2.4	996	92.6
290-293		4		3			4			3					6	3		23	2.1	1019	94.7
286-289				4			3			3					6	8		24	2.2	1043	96.9
282-285				1											1	2		4	0.4	1047	97.3
278-281		2		2			2								2	2		10	0.9	1057	98.2
274-277		1					1								5	4		11	1.0	1068	99.3
270-273		1					1								1			3	0.3	1071	99.5
266-269		1														2		3	0.3	1074	99.8
262-265																1		1	0.1	1075	99.9
258-261															1			1	0.1	1076	100.0
Total	62	38	64	98	41	29	96	38	98	56	68	92	108	56	74	52	6	1076			

Note: Underlined values are the grand average.

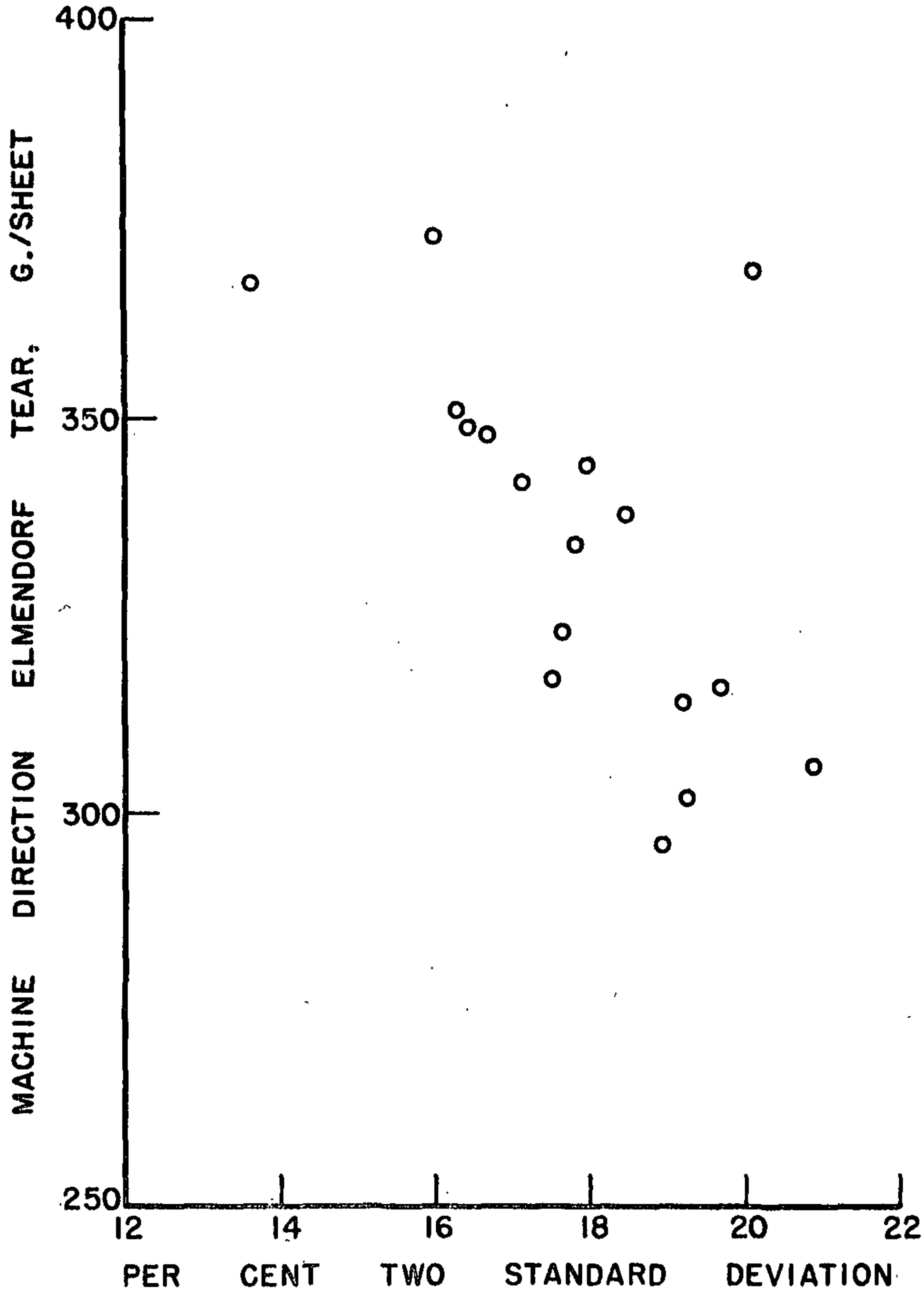

Figure 51. Relationship Between Machine Direction Elmendorf Tear and Within-Reel Variability

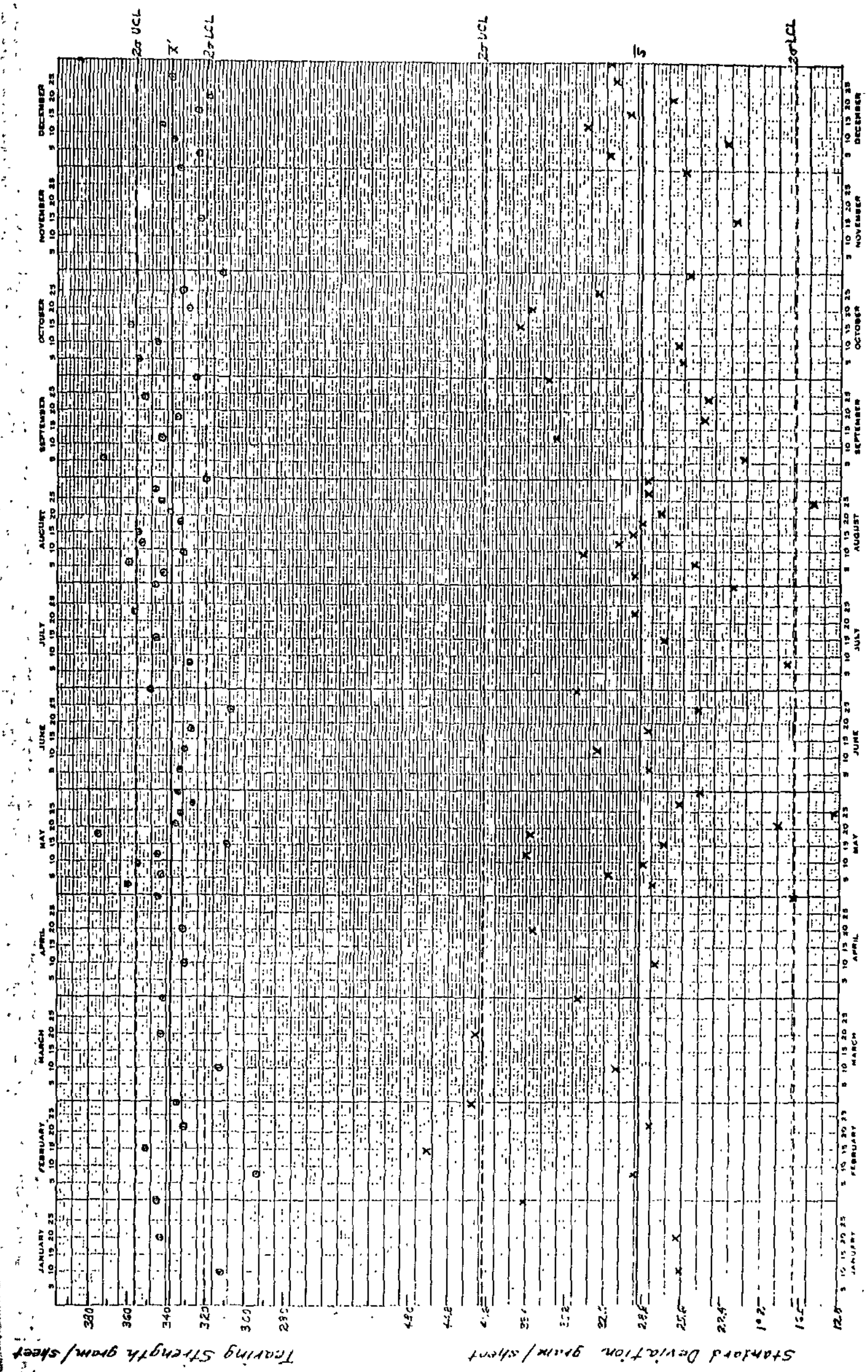

Figure 52

Machine Direction Elmendorf Tear--Mill A

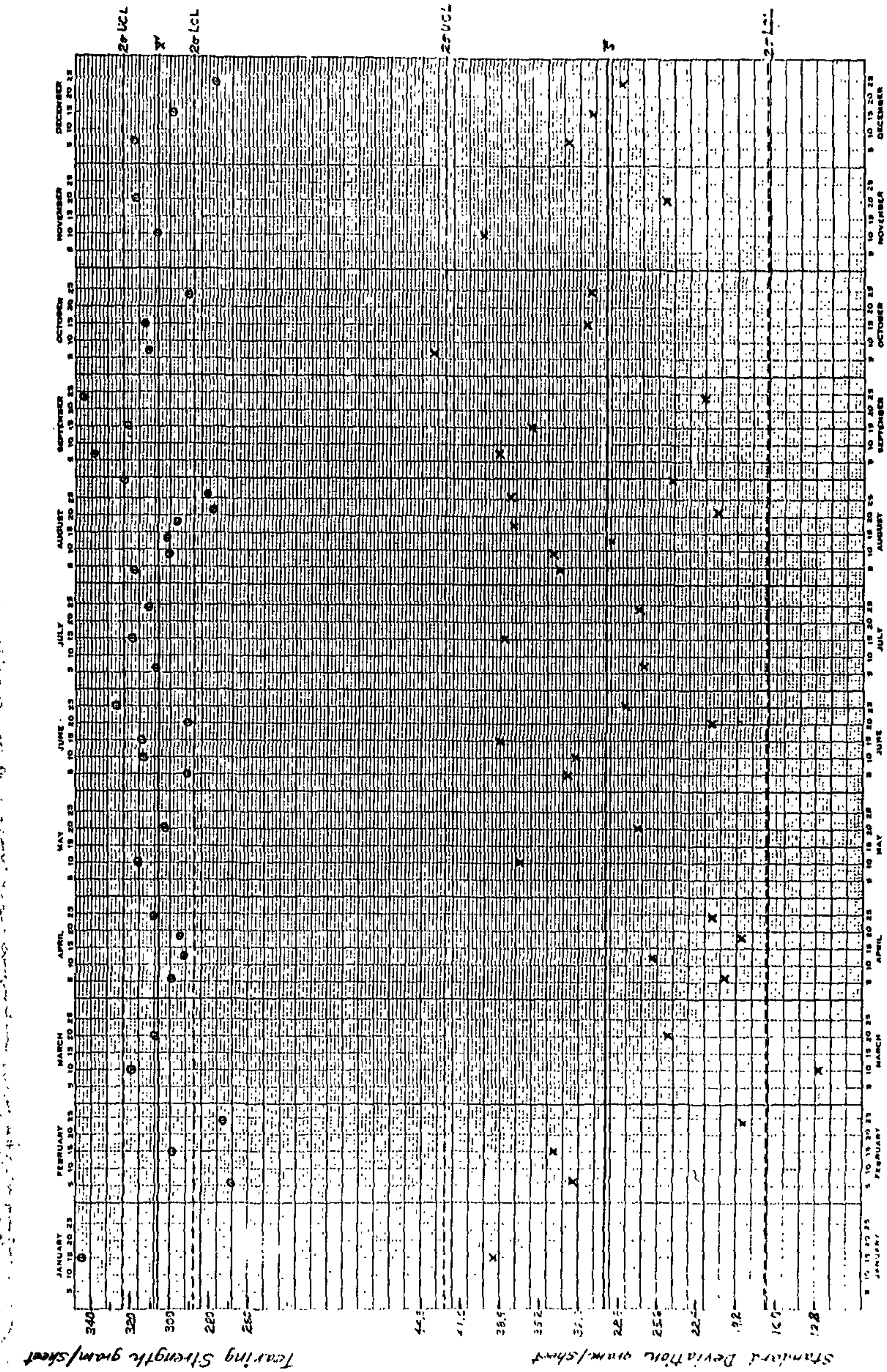

Figure 53

Machine Direction Elmendorf Tear--Mill B

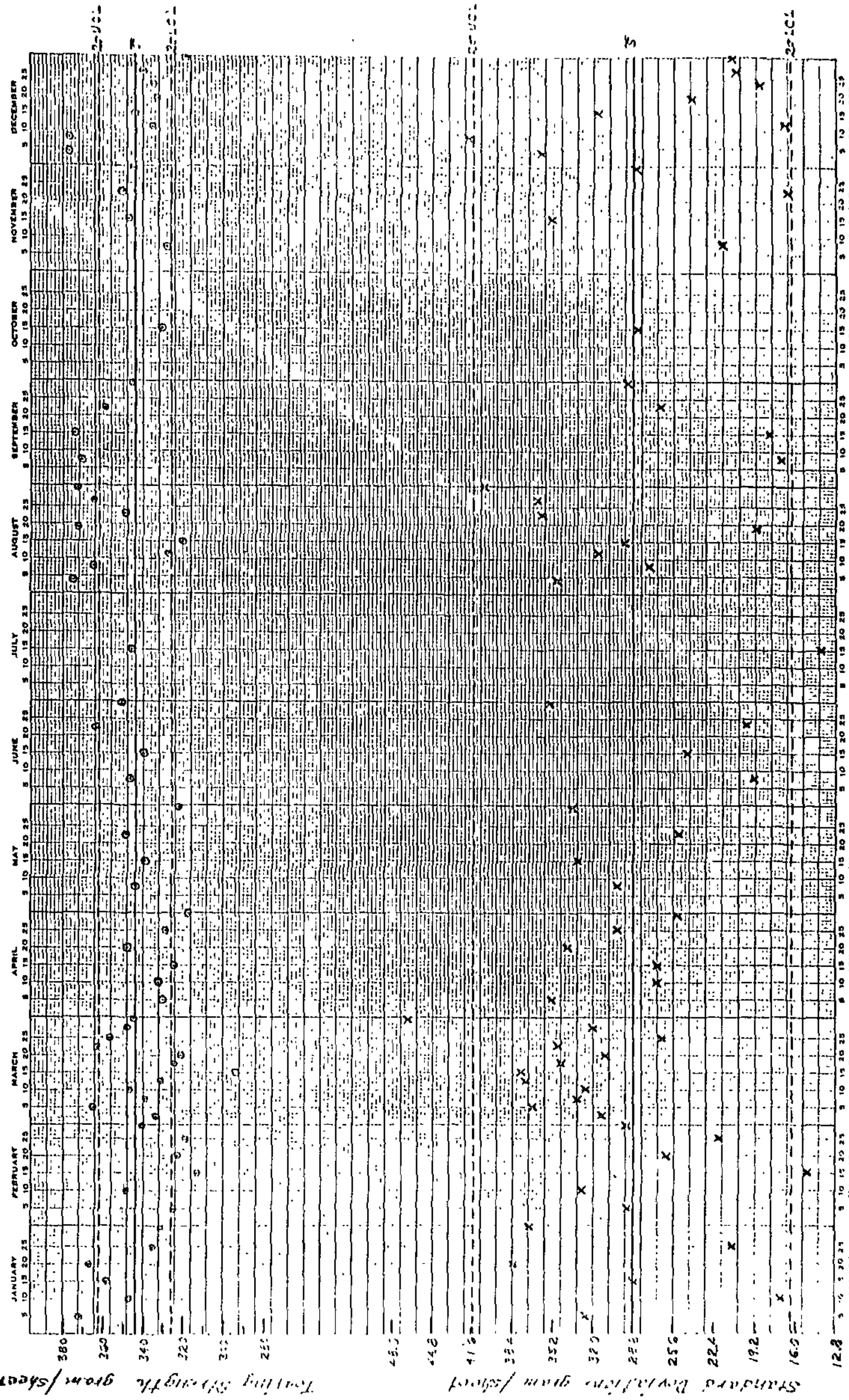

Figure 54

Machine Direction Elmendorf Tear—Mill C

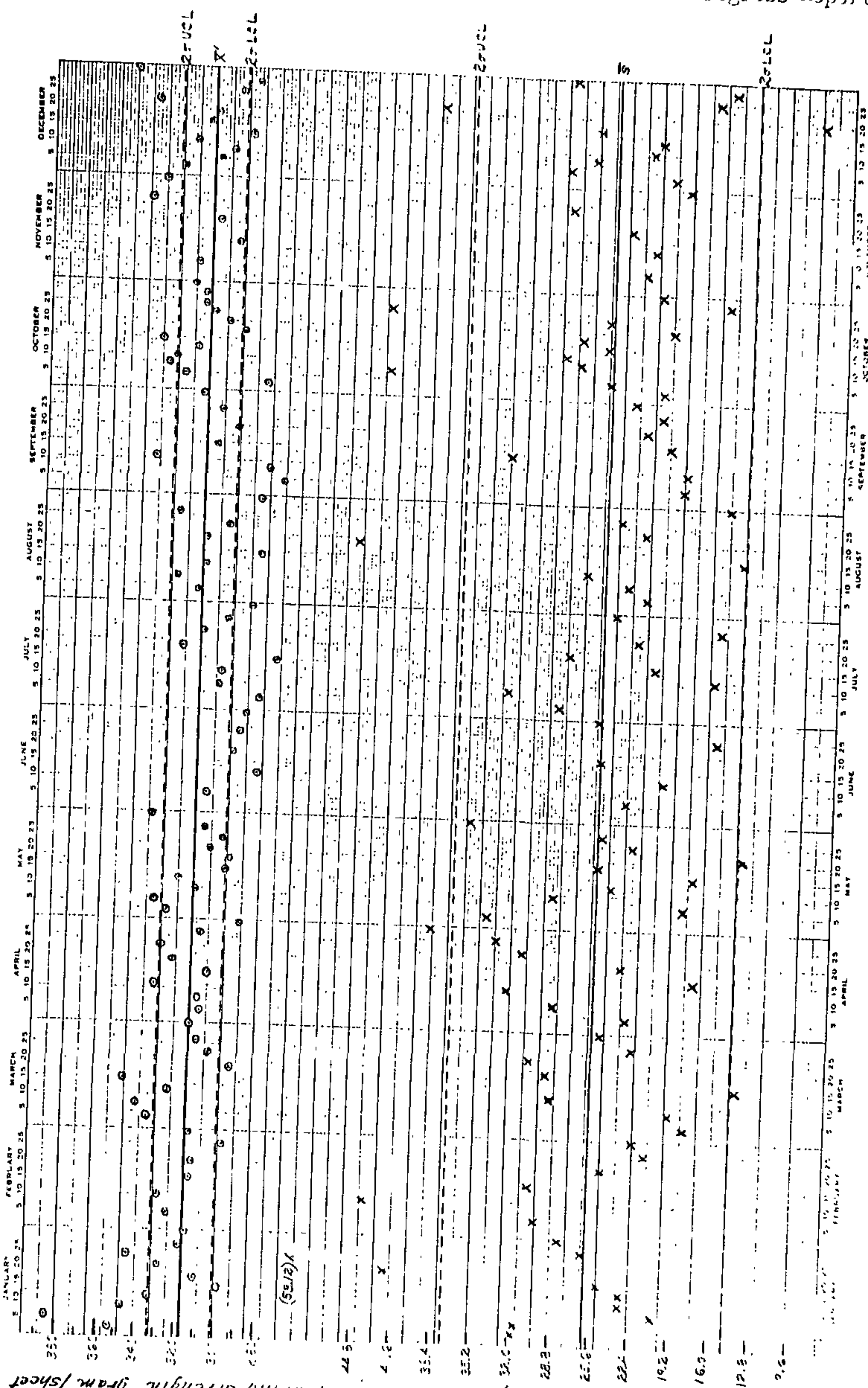

Figure 55

Machine Direction Elmendorf Tear--Mill D

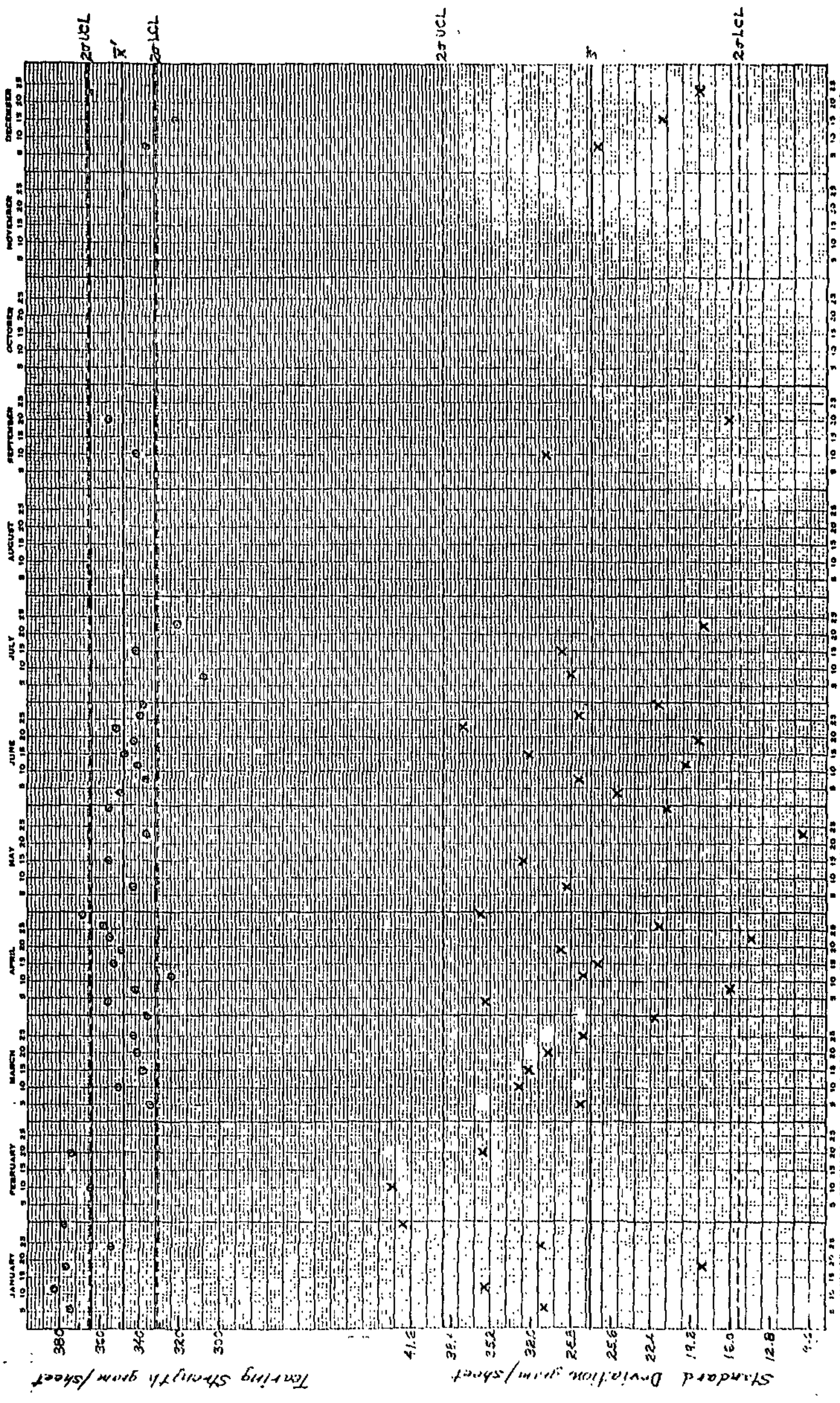

Figure 56

Machine Direction Elmendorf Tear--Mill E

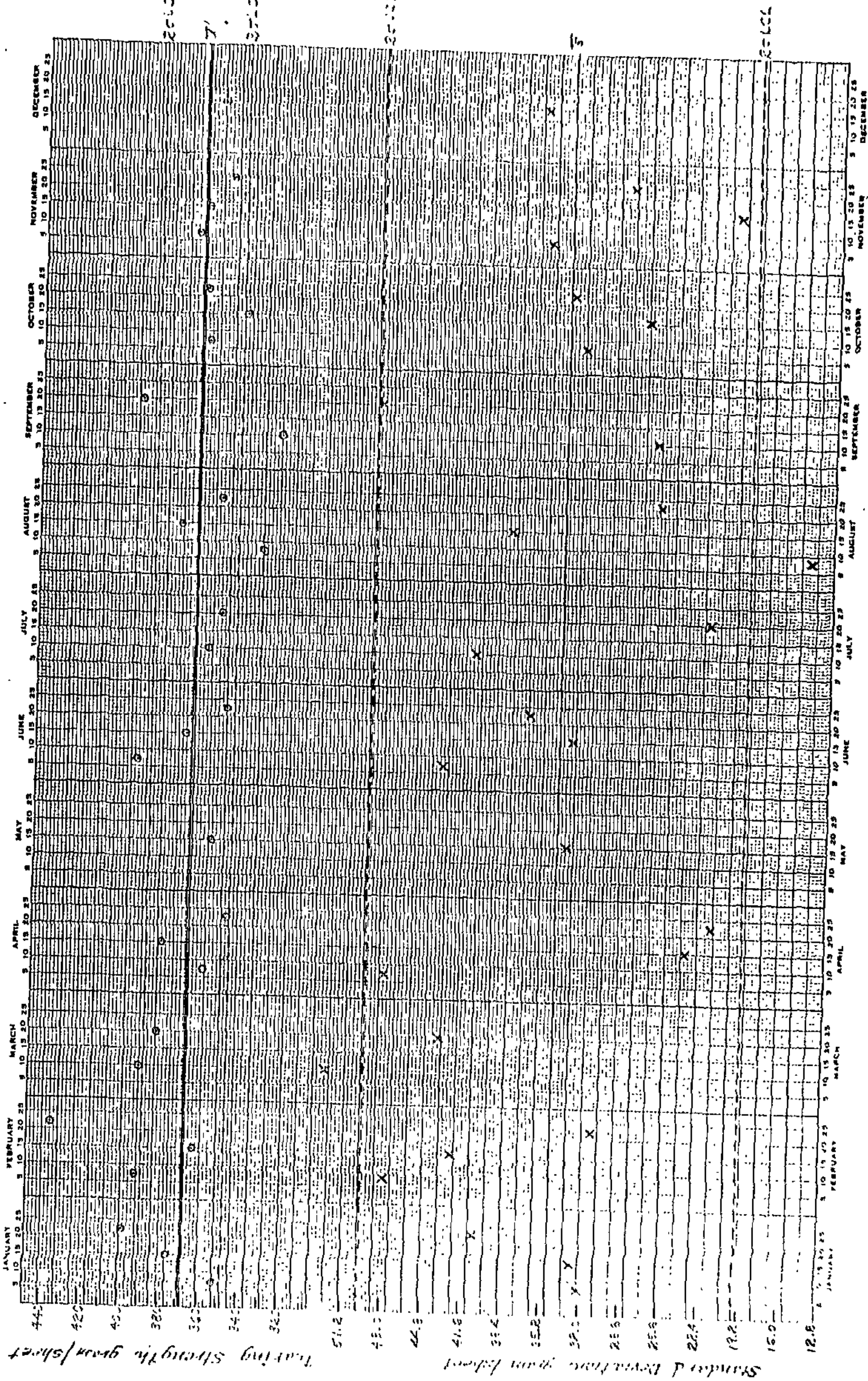

Figure 57

Machine Direction Elmendorf Tear--Mill F

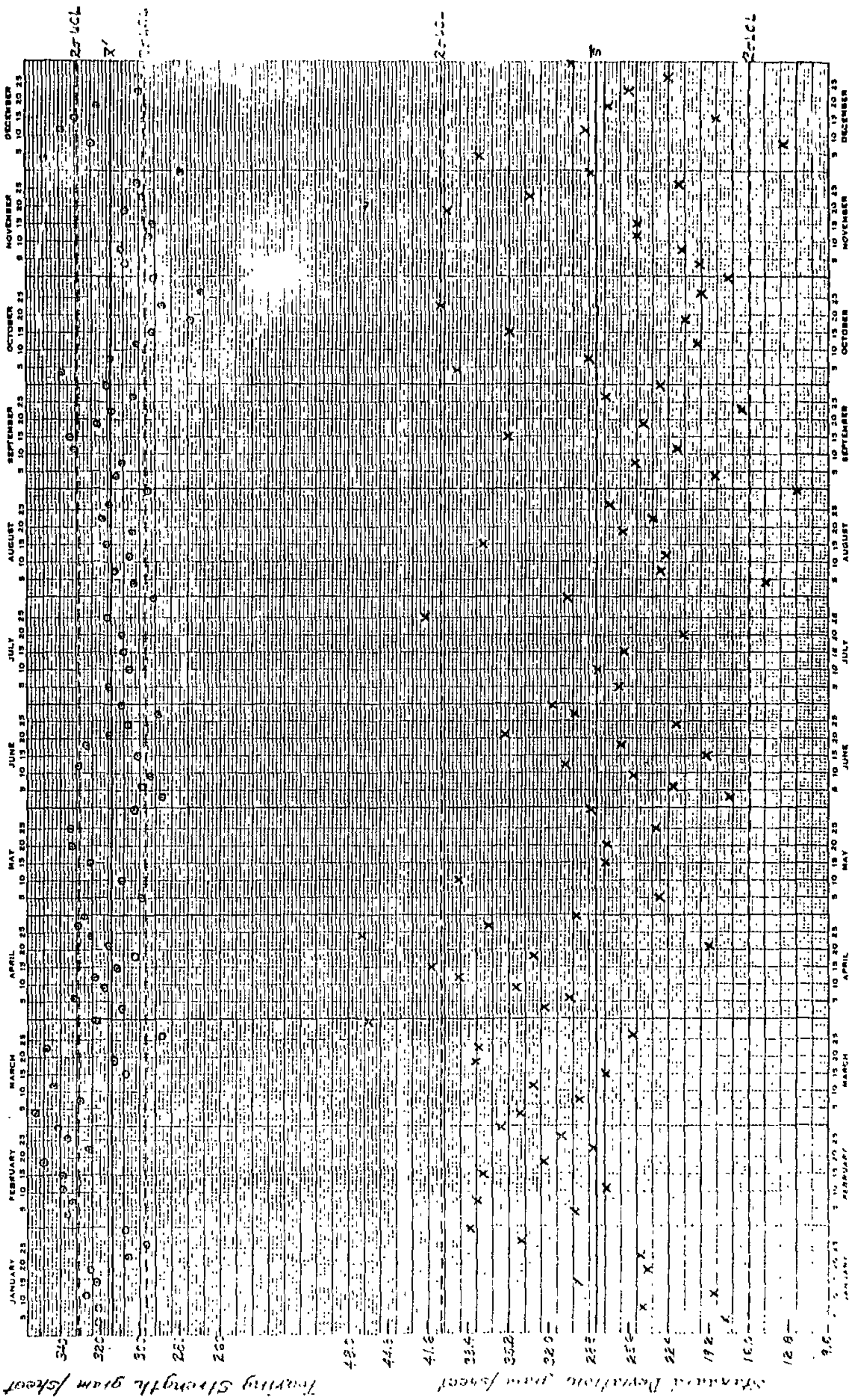

Figure 58

Machine Direction Elmendorf Tear--Mill G

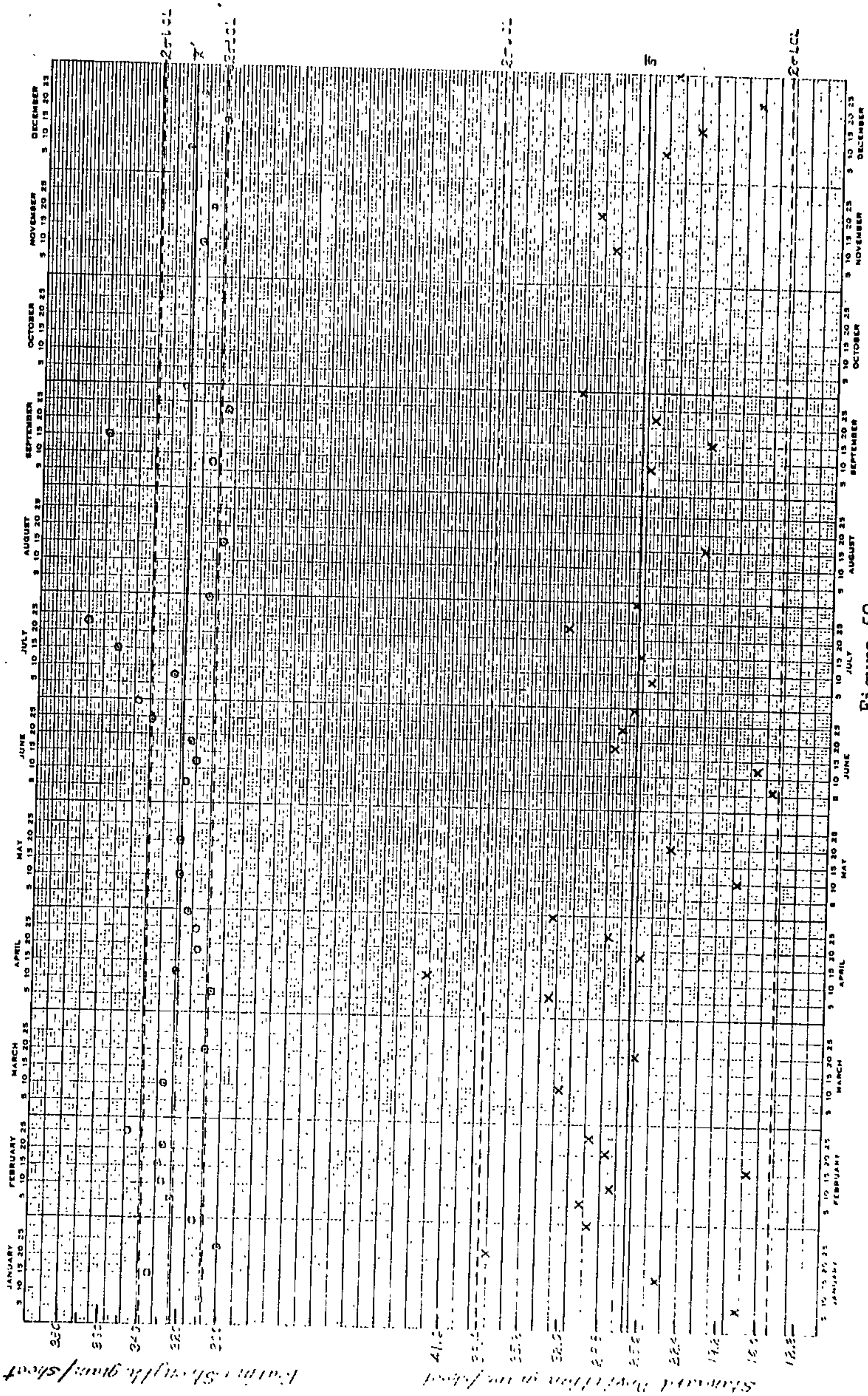

Figure 59

Machine Direction Elmendorf Tear--Mill H

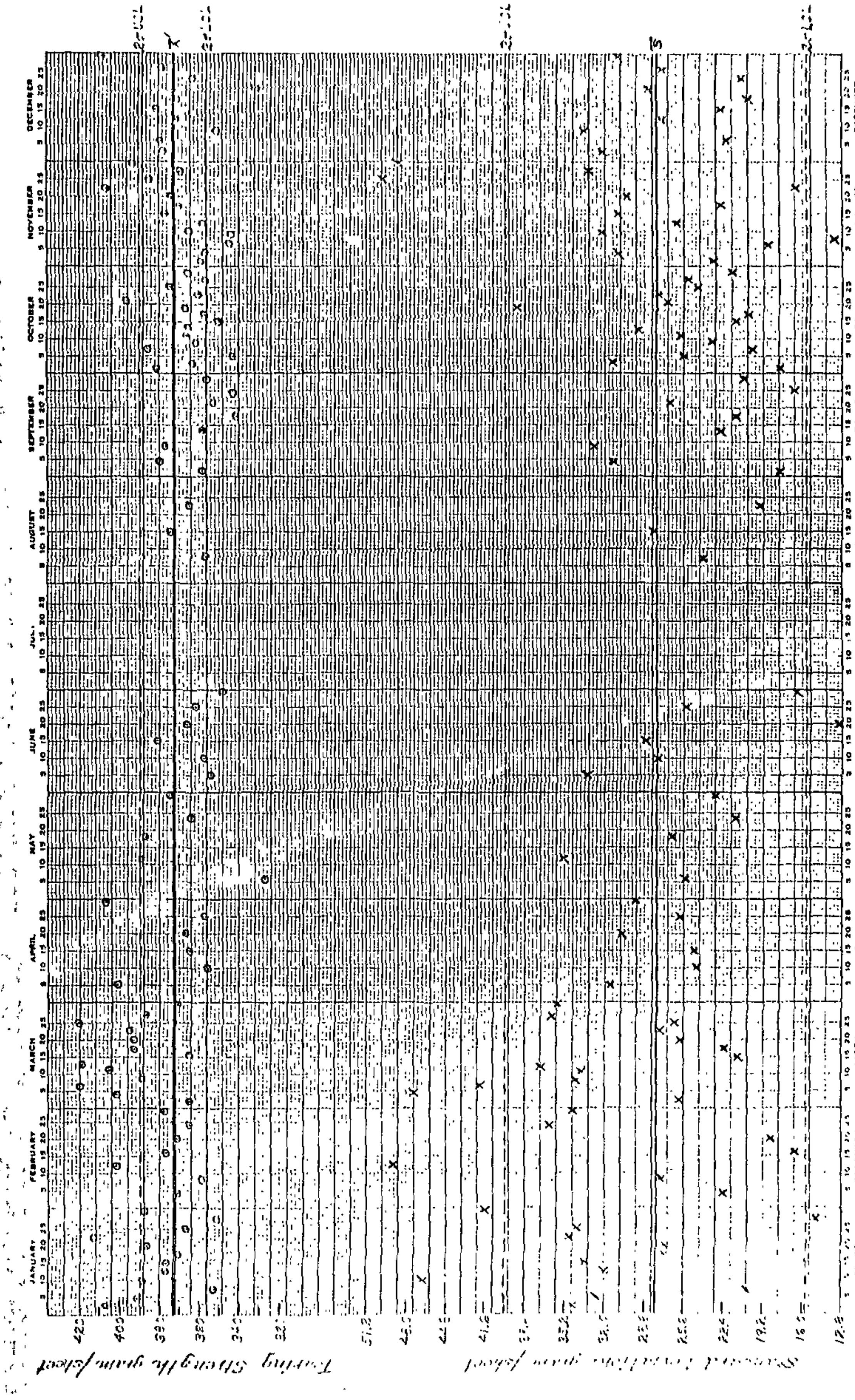

Figure 60

Machine Direction Elmendorf Tear--Mill I

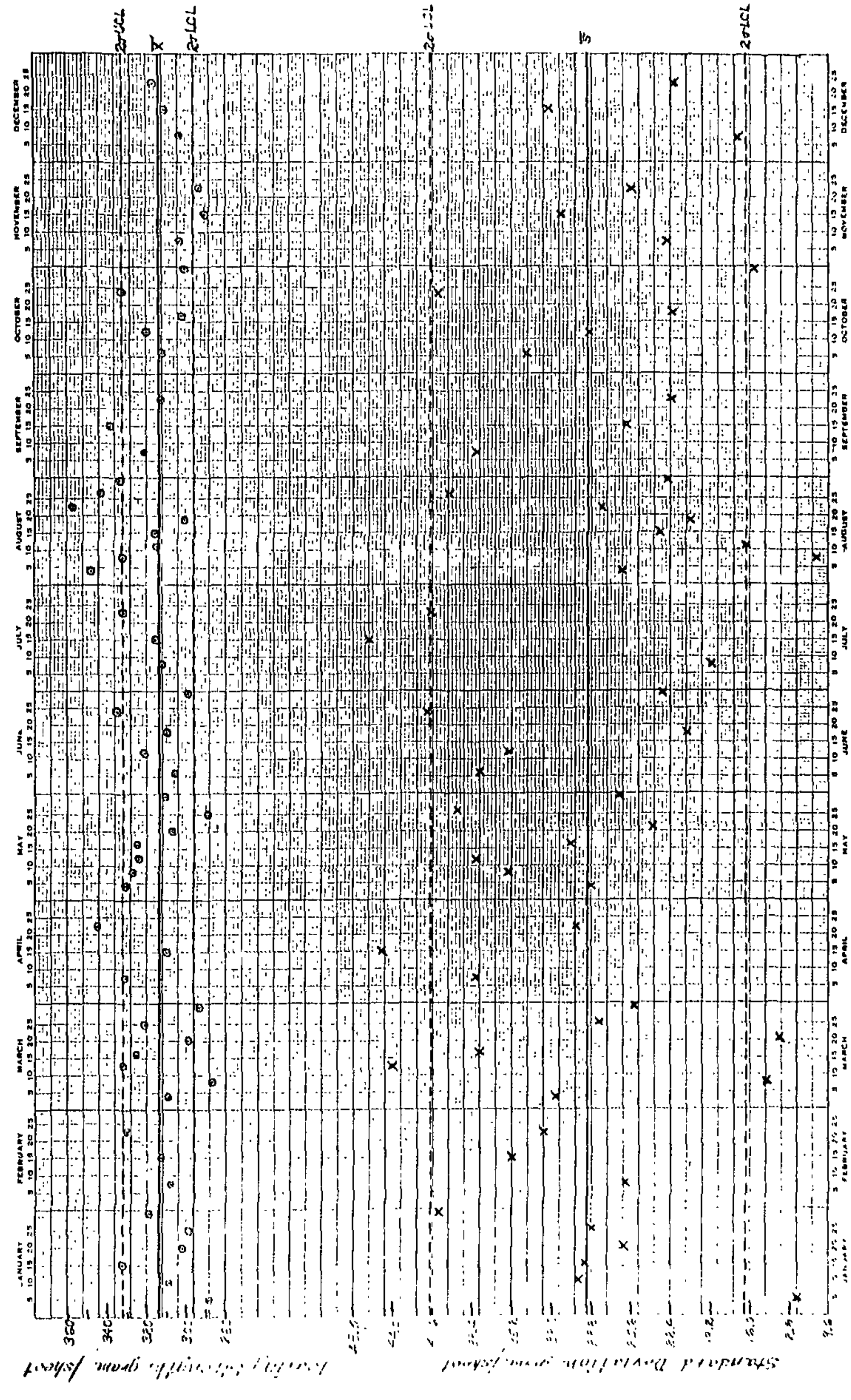

Figure 61

Machine Direction Elmendorf Tear--Mill J

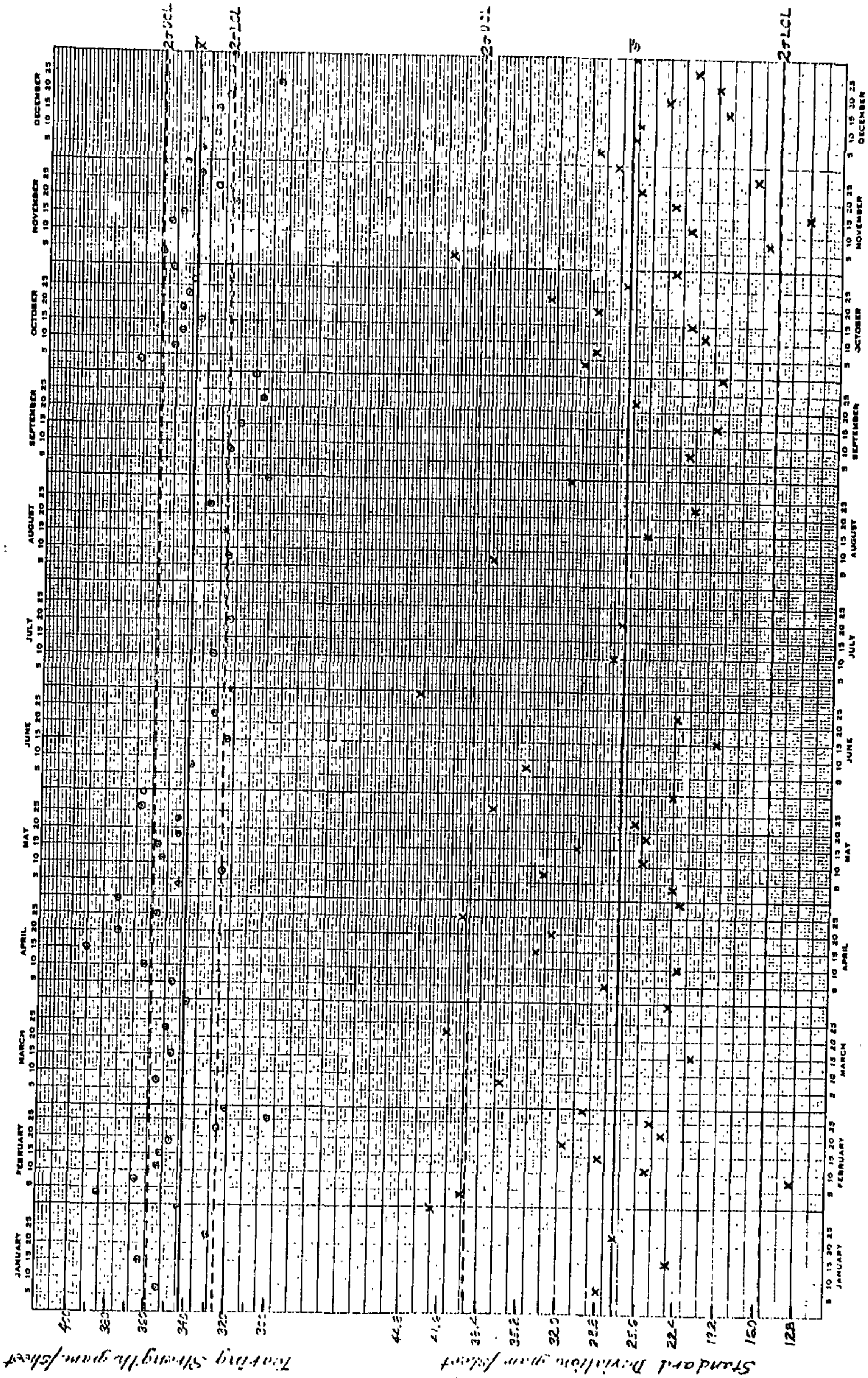

Figure 62

Machine Direction Elmendorf Tear—Mill K

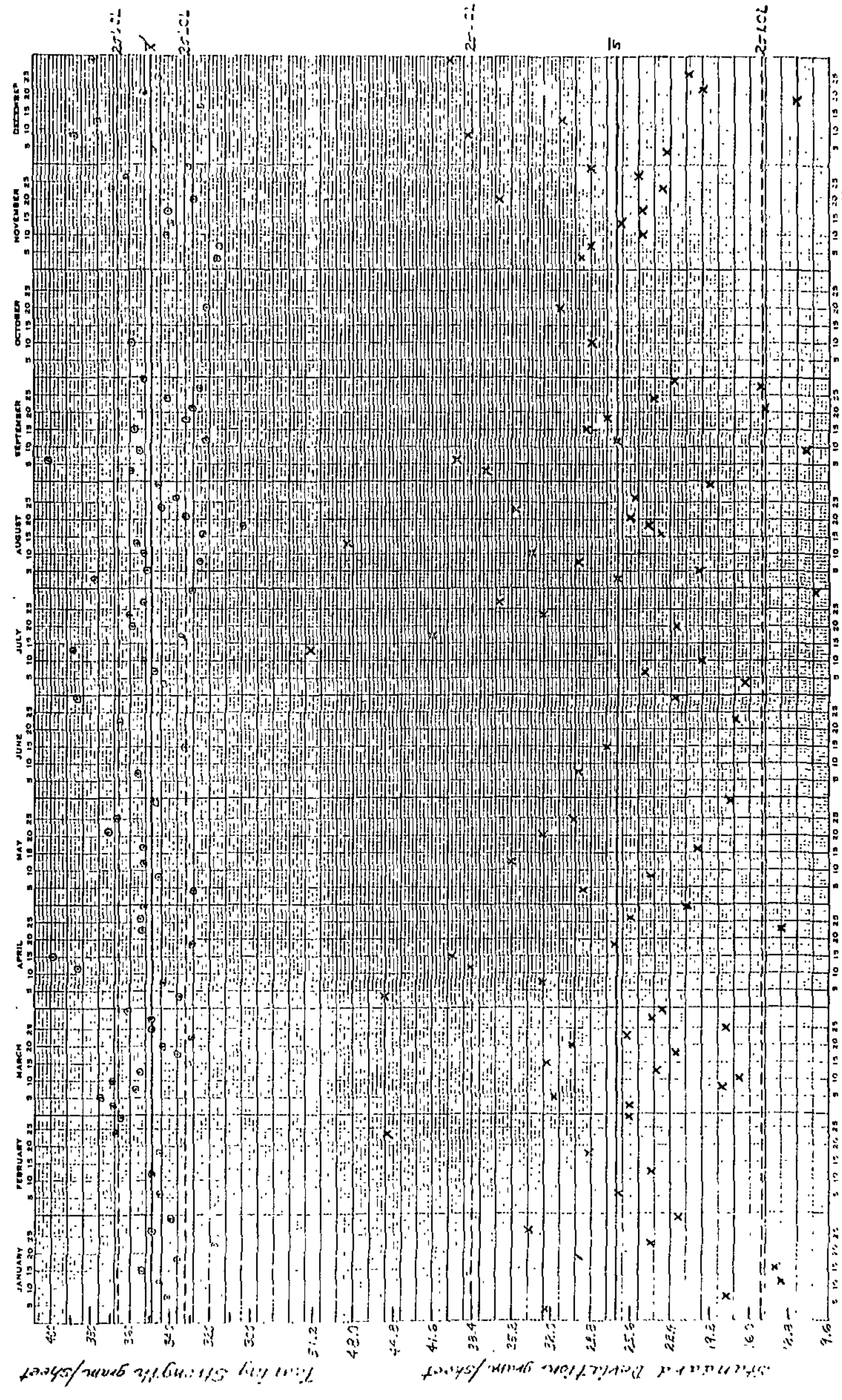

Figure 63

Machine Direction Elmendorf Tear--Mill L

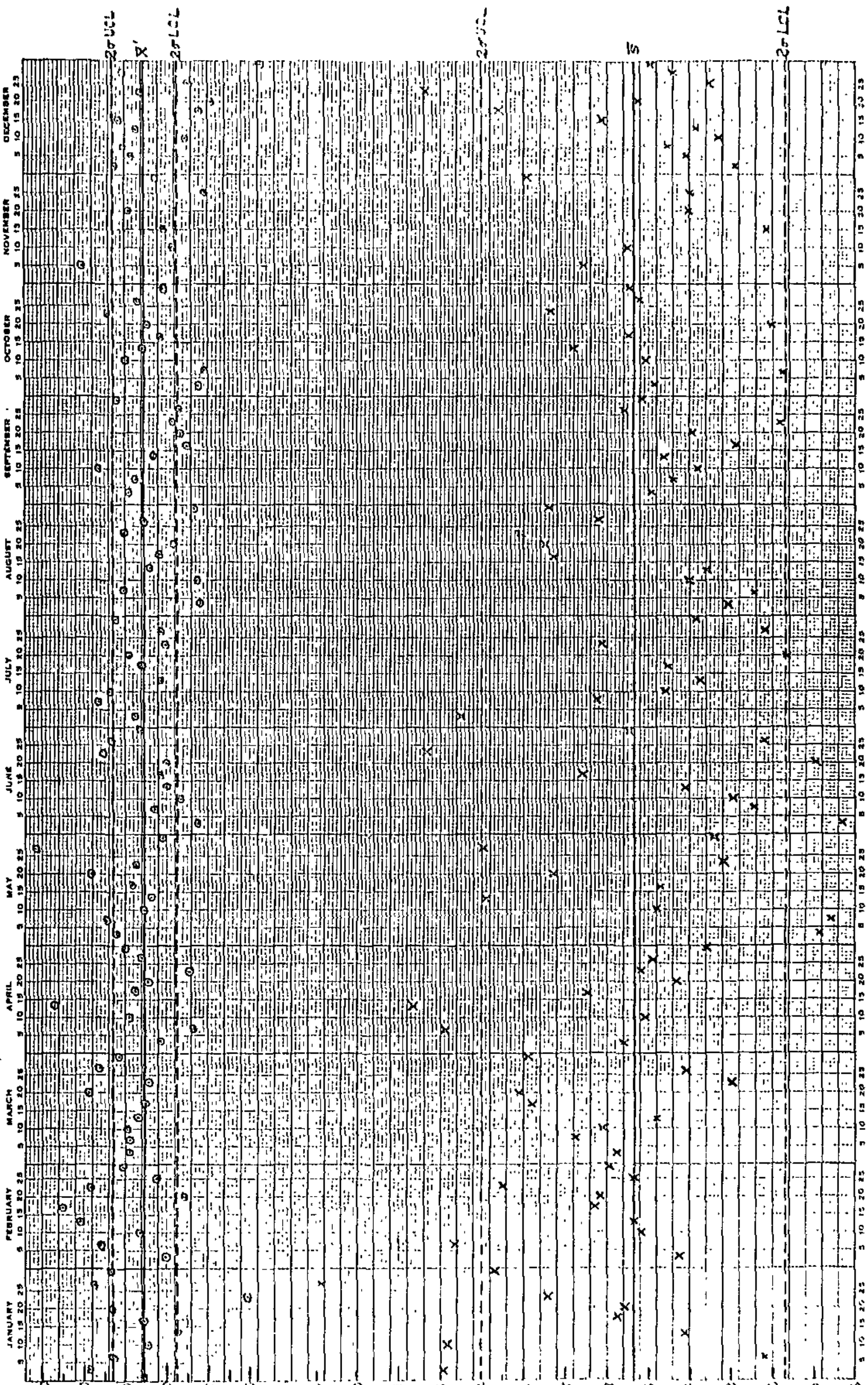

Figure 64

Machine Direction Elmendorf Tear--Mill M

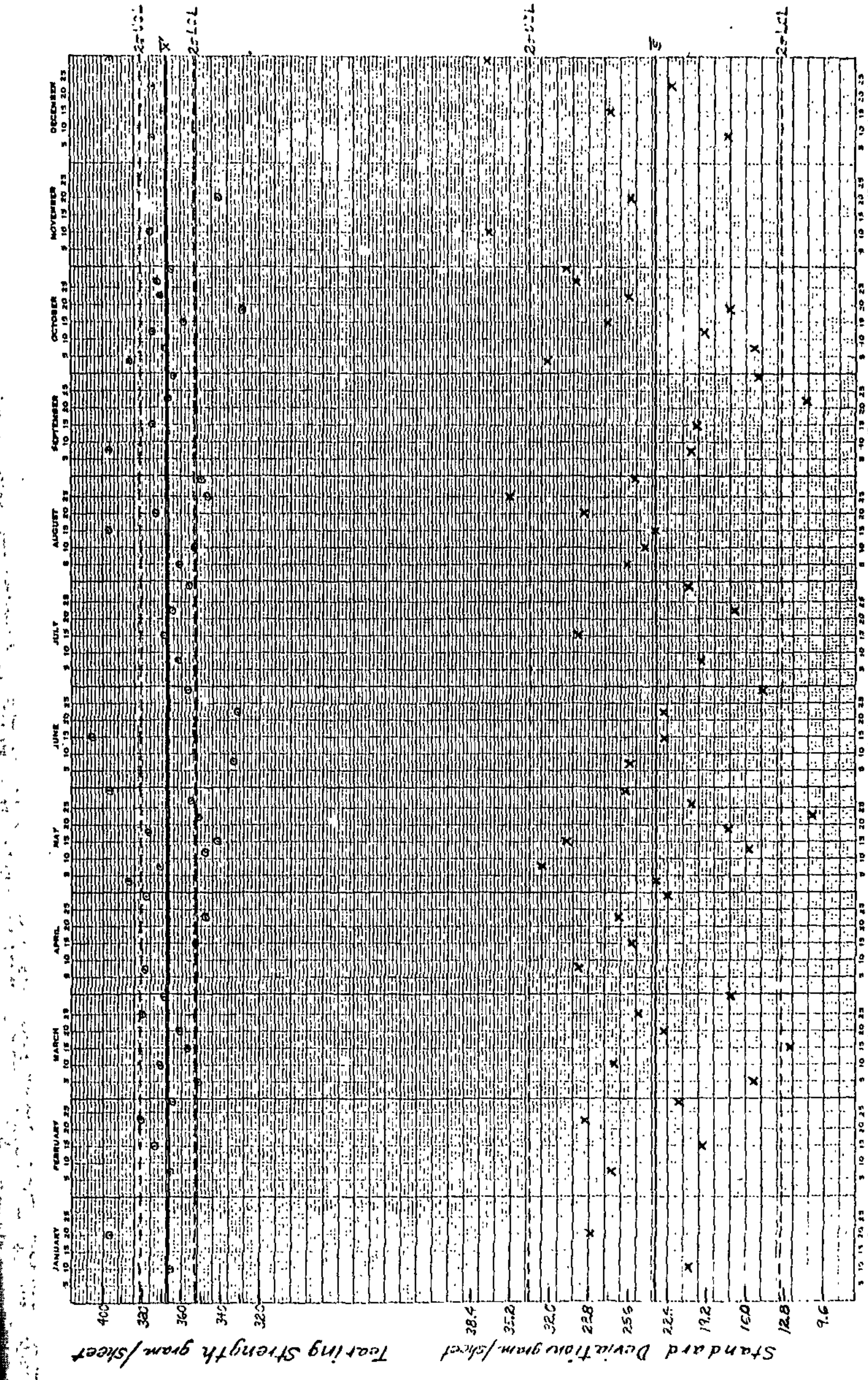

Figure 65

Machine Direction Elmendorf Tear--Mill N

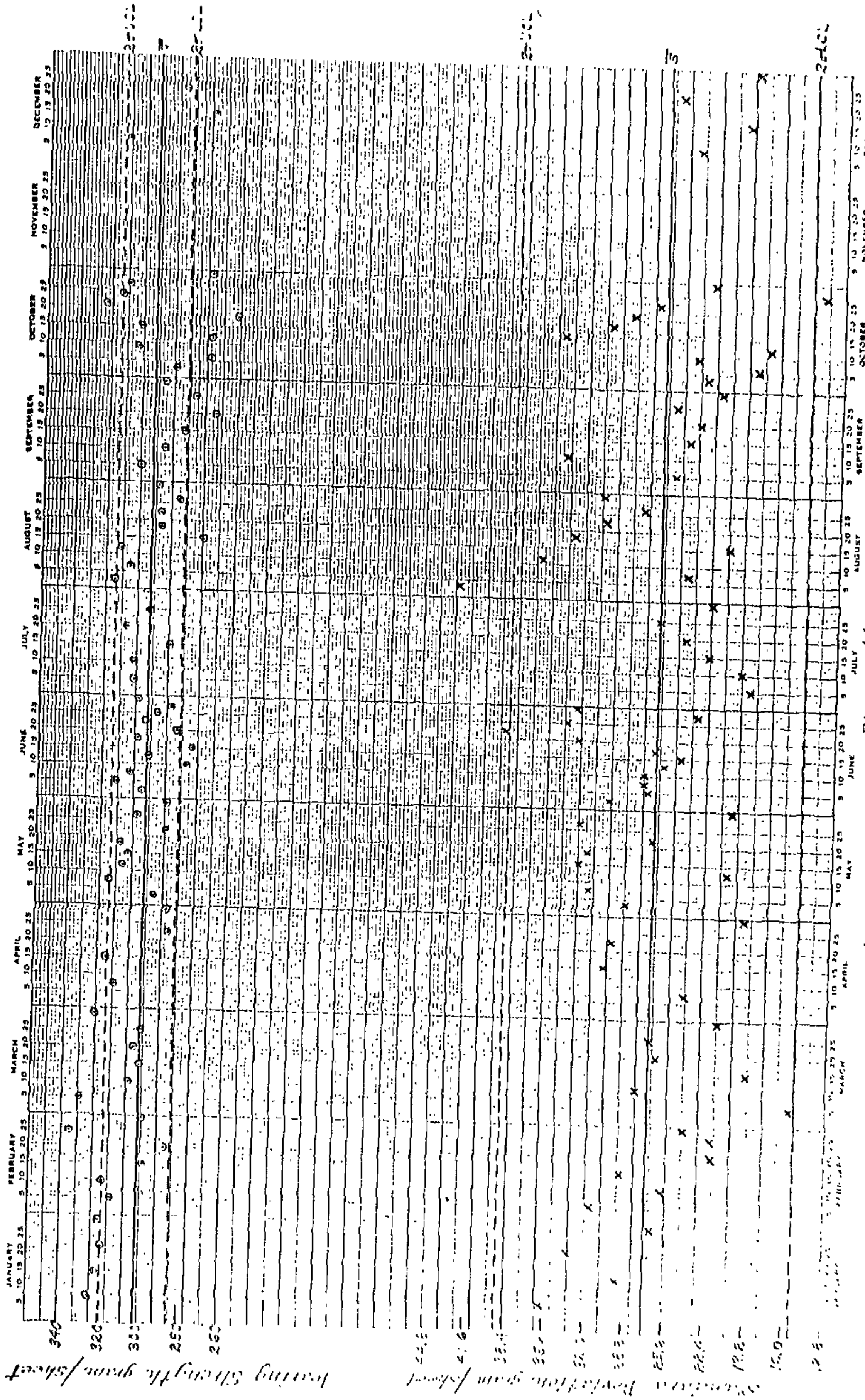

Figure 66
Machine Direction Elmendorf Tear—Mill O

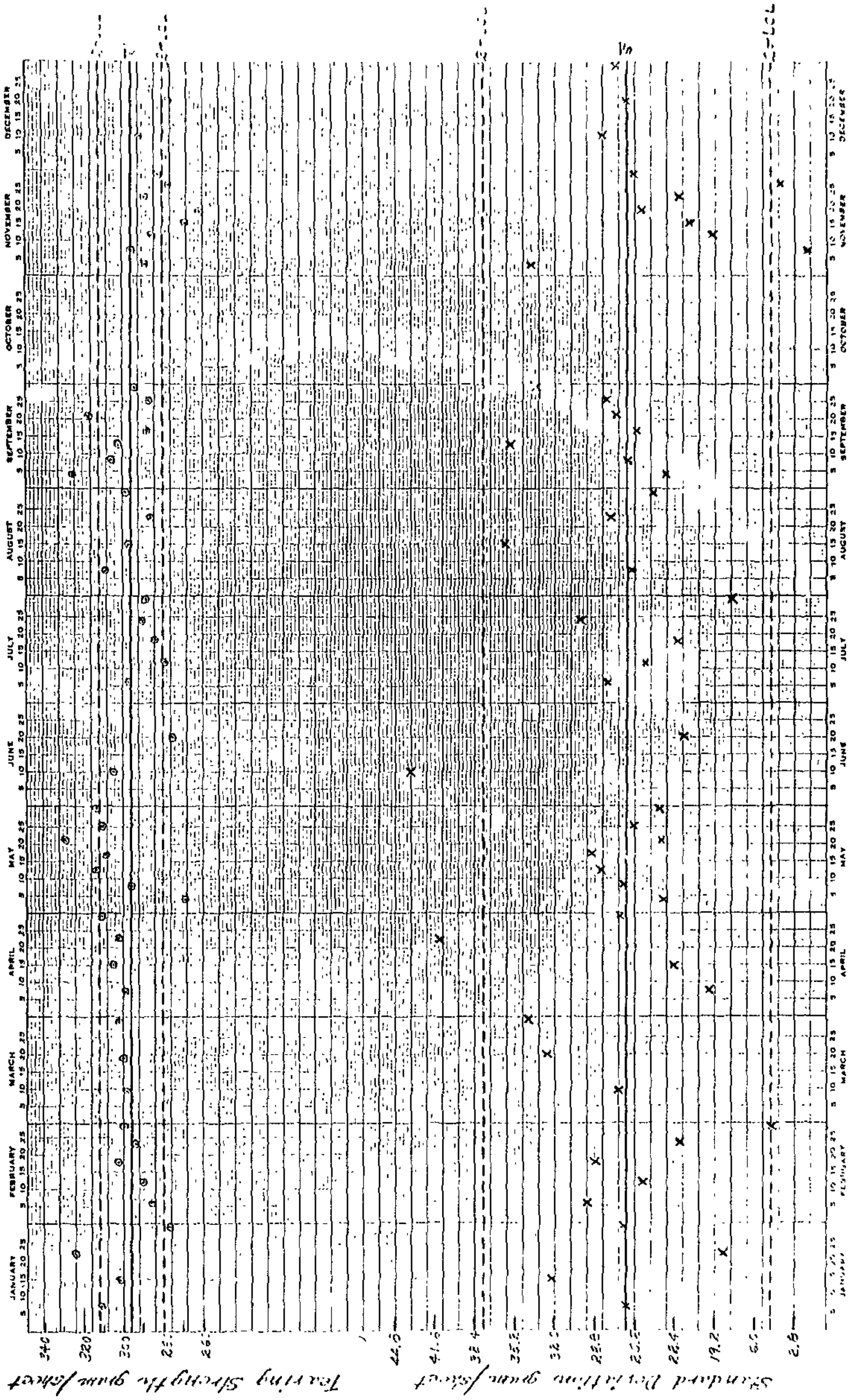

Figure 67

Machine Direction Elmendorf Tear--Mill P

ELMENDORF TEAR--CROSS-MACHINE DIRECTION

The comparisons of within and between reel variability are summarized in Table XI and the frequency distribution of the reel averages are tabulated in Table XII. In Table XI it may be noted that the within reel variability expressed in terms of per cent two standard deviation ranges from 10.94% for Mill D to 14.50% for Mill I. The between-reel variances ranged from 6.13% for Mill J to 11.65% for Mill K. The composite averages for the group are tabulated below for the in and across machine tear tests in terms of per cent standard error.

	Within-reel Variability, % two standard error	Between-reel Variability, % two standard error	Ratio of Between to Within-reel Variability
Elmendorf tear--In (N=12)	5.21	10.75	2.1
Elmendorf tear--Ac (N=12)	3.73	8.18	2.2

As may be noted above, somewhat lower within and between-reel variabilities were obtained for the cross-machine direction. It may be noted, however, that the ratios of between to within reel variability were nearly equal.

The cross-machine direction tear control charts are summarized in Figures 68 through 83. As in the case of the charts for the other properties it may be noted that the reel standard deviations appear to be substantially in control. The reel averages fall outside the control limits more frequently; however, a number of mills exhibit reasonable control of the reel averages also (see Mills J and P, for example).

TABLE XI

COMPARISON OF VARIABILITY WITHIN AND BETWEEN REELS OF CROSS MACHINE DIRECTION ELMENDORF TEARING STRENGTH BY MILLS

Mill	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	Composite
Within Reel																		
No. of samples	62	38	64	98	41	29	96	38	98	56	68	92	108	56	74	52	6	1076
Grand av., $\bar{X}'$	365	358	379	369	392	379	363	361	406	375	352	386	378	378	370	347	375	374
Av. standard deviation, $\bar{s}$	21.984	22.672	23.072	18.896	20.640	25.648	21.248	21.024	27.552	24.848	21.744	24.096	23.104	22.288	21.680	20.816	20.944	22.606
Estimated population standard deviation, σ	23.488	24.224	24.656	20.192	22.048	27.408	22.704	22.464	29.440	26.544	23.232	25.744	24.688	23.808	23.168	22.240	22.384	24.154
Per cent two std. deviation	12.87	13.53	13.01	10.94	11.25	14.46	12.51	12.45	14.50	14.16	13.20	13.34	13.06	12.60	12.52	12.82	11.94	12.92
Two standard error, $2\sigma/\sqrt{n}$	13.562	13.986	14.236	11.658	12.730	15.824	13.108	12.970	16.998	15.326	13.414	14.864	14.254	13.746	13.376	12.840	12.524	13.946
Per cent two standard error	3.72	3.91	3.76	3.16	3.25	4.18	3.61	3.59	4.19	4.09	3.81	3.86	3.77	3.64	3.62	3.70	3.34	3.73
Two S.E. limits about $\bar{X}'$	351-379	344-372	365-393	357-381	379-405	363-395	350-376	348-374	389-423	360-390	339-365	371-401	364-392	364-392	357-383	334-360	362-388	360-388
Two S.E. limits about $\bar{s}$	12.400-31.568	12.784-32.560	13.008-33.136	10.656-27.136	11.632-29.648	14.464-36.832	11.984-30.512	11.856-30.192	12.016-39.568	14.016-35.680	12.240-31.248	13.584-34.608	12.960-33.184	12.576-32.000	12.224-31.136	11.744-33.088	11.808-30.080	12.745-32.467
Between Reel																		
Two standard error $2\sigma_1/\sqrt{n}$	28.8	35.0	27.4	28.4	32.8	42.2	28.6	24.0	32.6	23.0	41.0	39.6	29.0	26.0	33.2	22.2	25.8	30.6
Per cent two standard error	7.89	9.78	7.23	7.70	8.37	11.13	7.88	6.65	8.03	6.13	11.65	10.26	7.67	6.88	8.97	6.40	6.88	8.18

TABLE XII

FREQUENCY DISTRIBUTION OF CROSS MACHINE DIRECTION ELMENDORF TEARING AVERAGES

Tearing Strength, grams/sheet	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	Total	Per Cent	Cumula-tive Total	Cumula-tive, %
444-447									1									1	0.1	1	0.1
440-443												1						1	0.1	2	0.3
436-439									3									3	0.3	5	0.5
432-435					1				2									3	0.3	8	0.7
428-431									3			1						4	0.4	12	1.1
424-427					2	1			3			2						8	0.8	20	1.9
420-423				1					9									10	0.9	30	2.8
416-419			1						7			4	1					13	1.2	43	4.0
412-415					1				11			3	1					16	1.5	59	5.5
408-411		1			4				6			2						13	1.2	72	6.7
404-407			3		2				6			7	3	1				22	2.0	94	8.7
400-403	1	1		1	3	2	1		14	1			1	2	2			29	2.7	123	11.4
396-399			3	4	3	1	1		8	1		4	1	2	2			30	2.8	153	14.2
392-395			3	2	3	2	1		6	2	3	8	7	7	3			47	4.4	200	18.6
388-391	2		6	1	4	2	2		6	4	1	7	13	2	2		2	54	5.0	254	23.6
384-387	3	2	10	2	4	2	4	1	5	6		11	14	6	4			74	6.9	328	30.5
380-383	7		3	6	3	3	3		3	6	3	12	9	3	8			69	6.4	397	36.9
376-379			9	9	6	4	2	3		7	4	8	13	3	12		1	81	7.5	478	44.4
372-375	5		6	8	1	2	10	3	3	4	4	4	11	9	8	1	1	80	7.4	558	51.9
368-371	10	6	3	11	3	5	12	5	1	6	2	4	14	9	4			95	8.8	653	60.7
364-367	6	3	10	21		1	12	3	1	10	4	4	8	3	4	3	1	94	8.7	747	69.4
360-363	7	5	3	10		1	11	6		5	4	3	2	5	5	3		70	6.5	817	75.9
356-359	5	2	1	4	1	2	12	5		2	4	1	2	2	4	5		53	4.9	870	80.9
352-355	6	3	1	9			9	4		2	4	1	1	2	4	7	1	54	5.0	924	85.9
348-351	2	4	2	6		1	6	3			6	1	1		3	6		41	3.8	965	89.7
344-347	2	2		1			3	2			2	2	3		5	3		25	2.3	990	92.0
340-343	2	3		1			1	1			2	2	2			11		25	2.3	1015	94.3
336-339	2	1		1			3	1			5		1		2	6		22	2.0	1037	96.4
332-335	2	1					1				9				1	2		16	1.5	1053	97.9
328-331		3						1			2				1	3		10	0.9	1063	98.8
324-327							2				4					1		7	0.7	1070	99.4
320-323											2					1		3	0.3	1073	99.7
316-319											3							3	0.3	1076	100.0
Total	62	38	64	98	41	29	96	38	98	56	68	92	108	56	74	52	6	1076			

Note: Underlined values are the grand average.

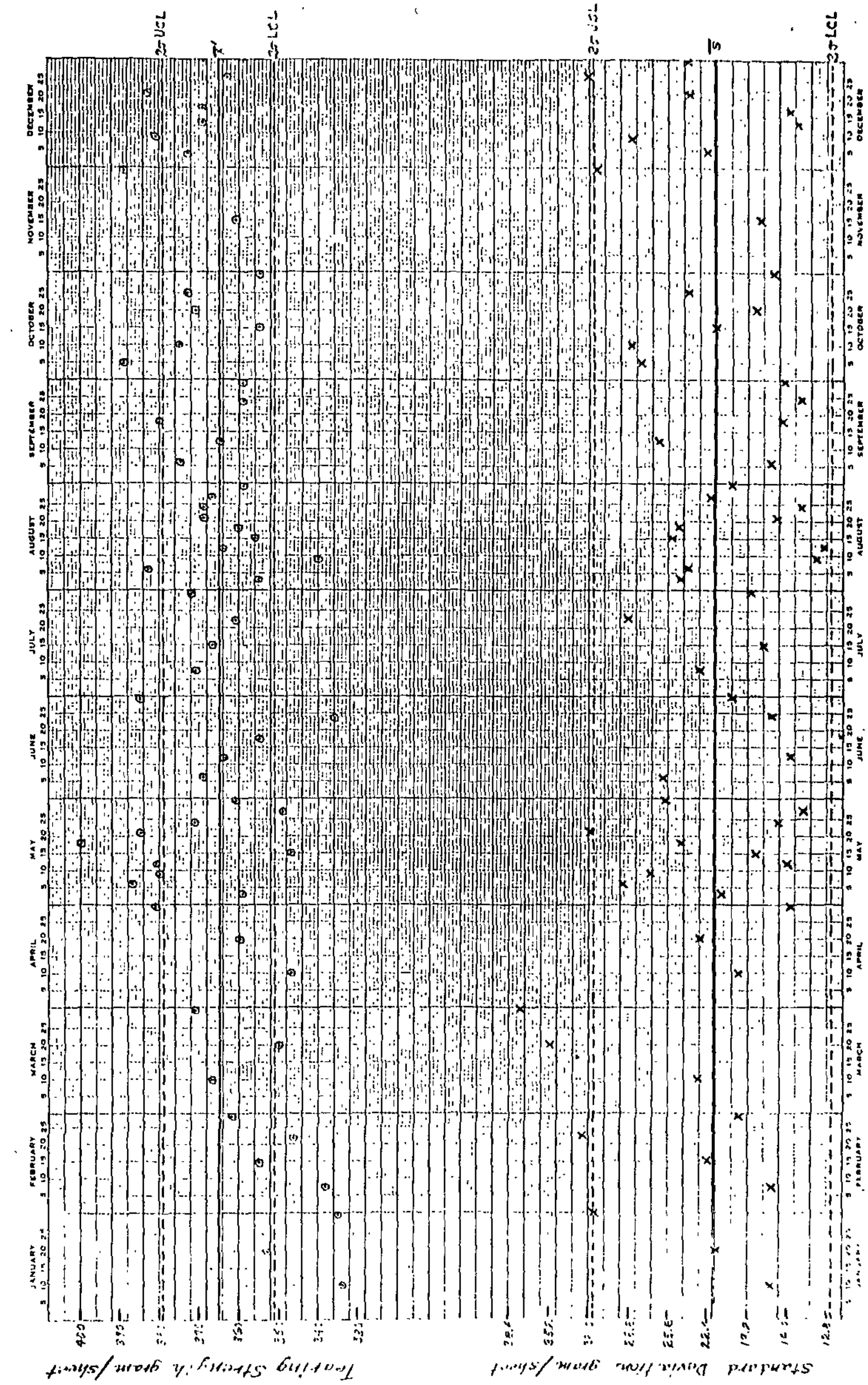

Figure 68

Cross-Machine Direction Elmendorf Tear--Mill A

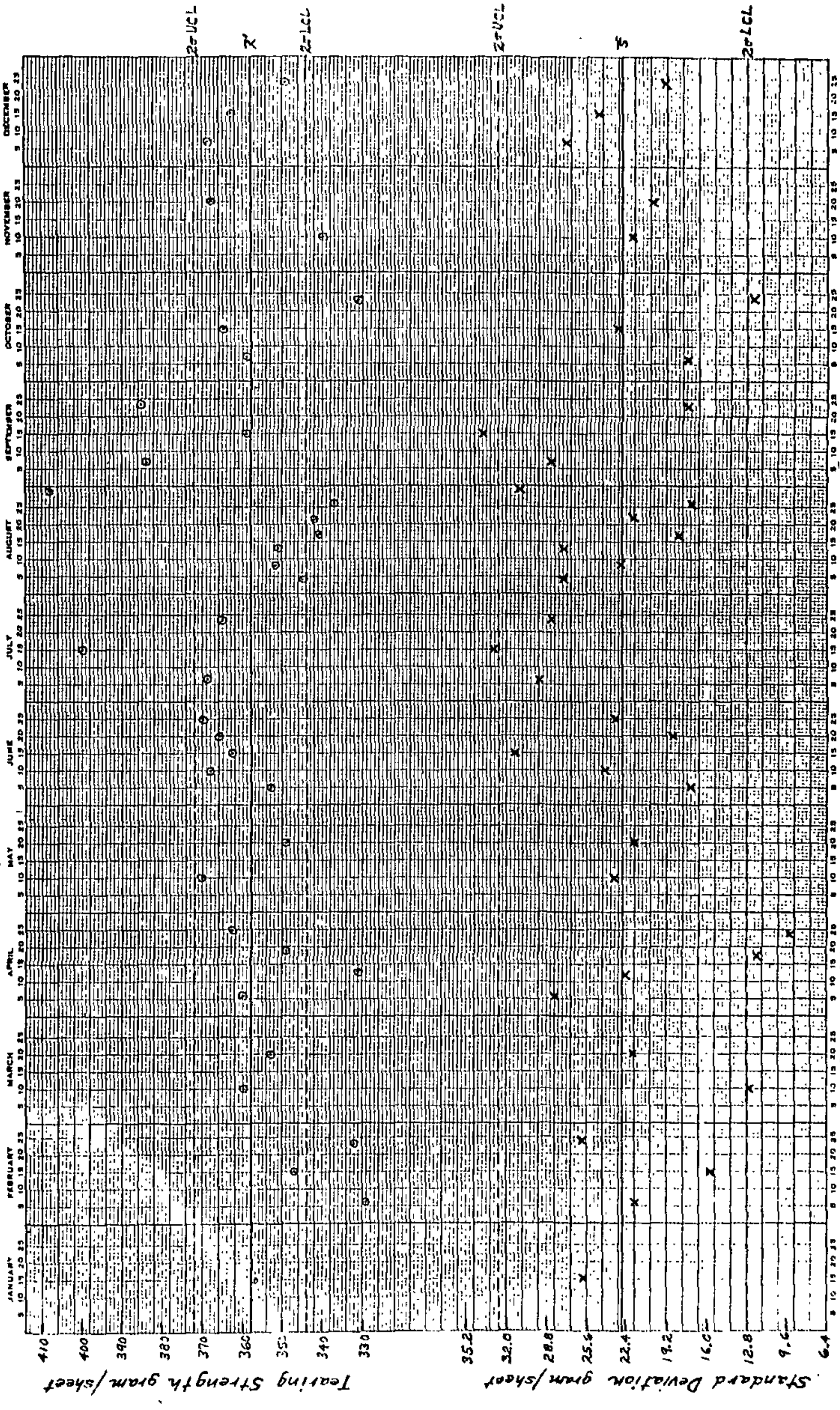

Figure 69

Cross-Machine Direction Elmendorf Tear--Mill B

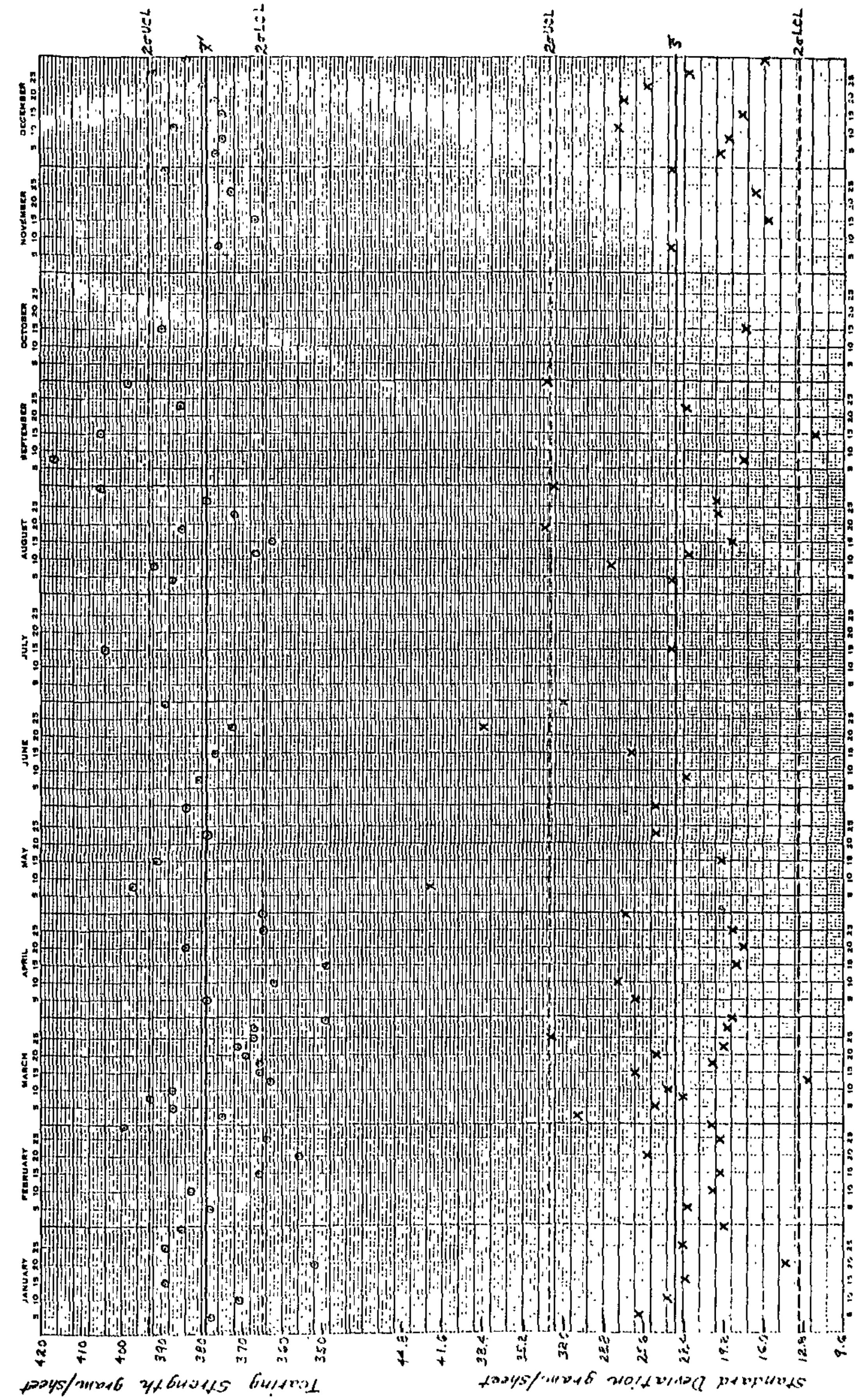

Figure 70

Cross-Machine Direction Elmendorf Tear--Mill C

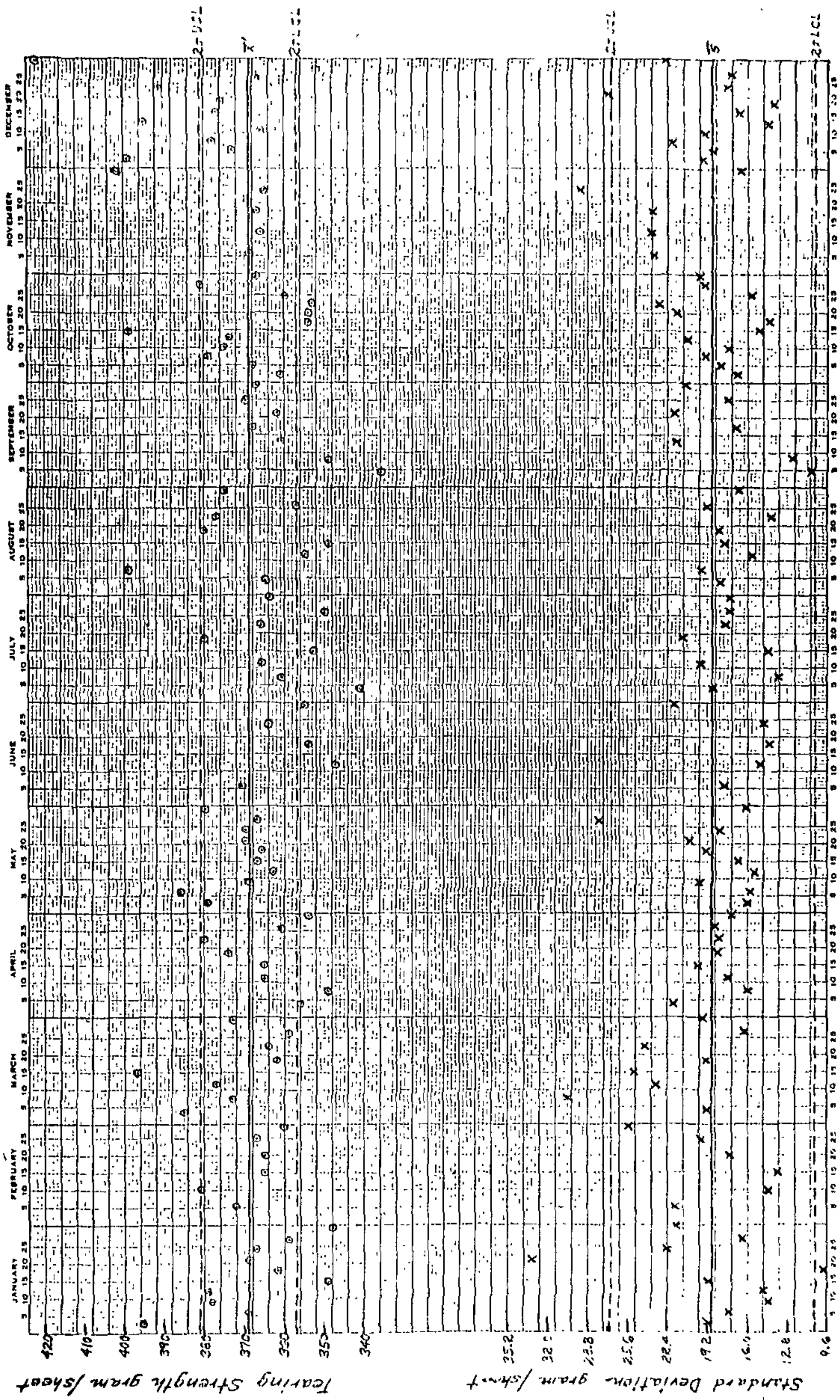

Figure 71

Cross-Machine Direction Elmendorf Tear--Mill D

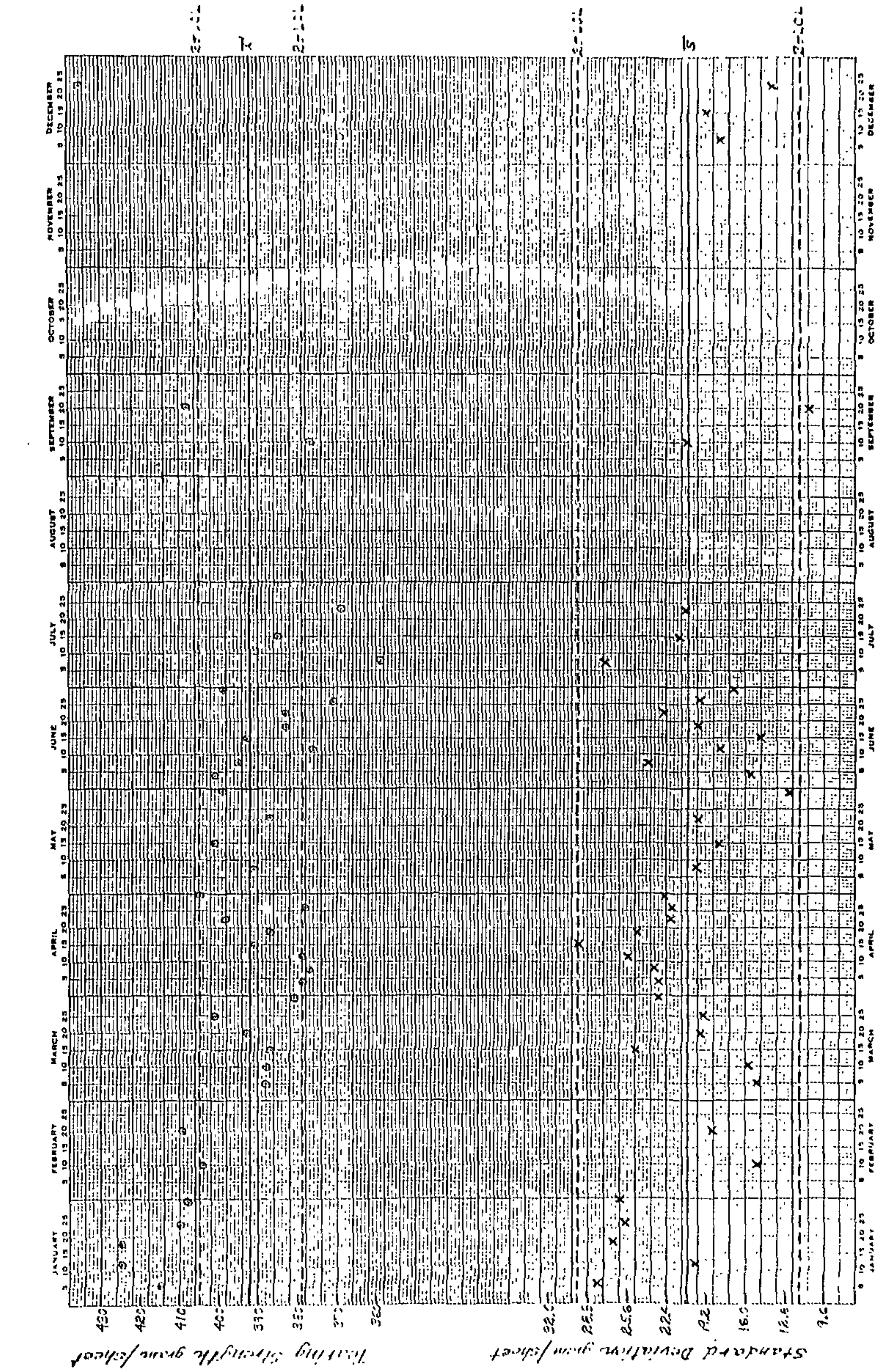

Figure 72

Cross-Machine Direction Elmendorf Tear--Mill E

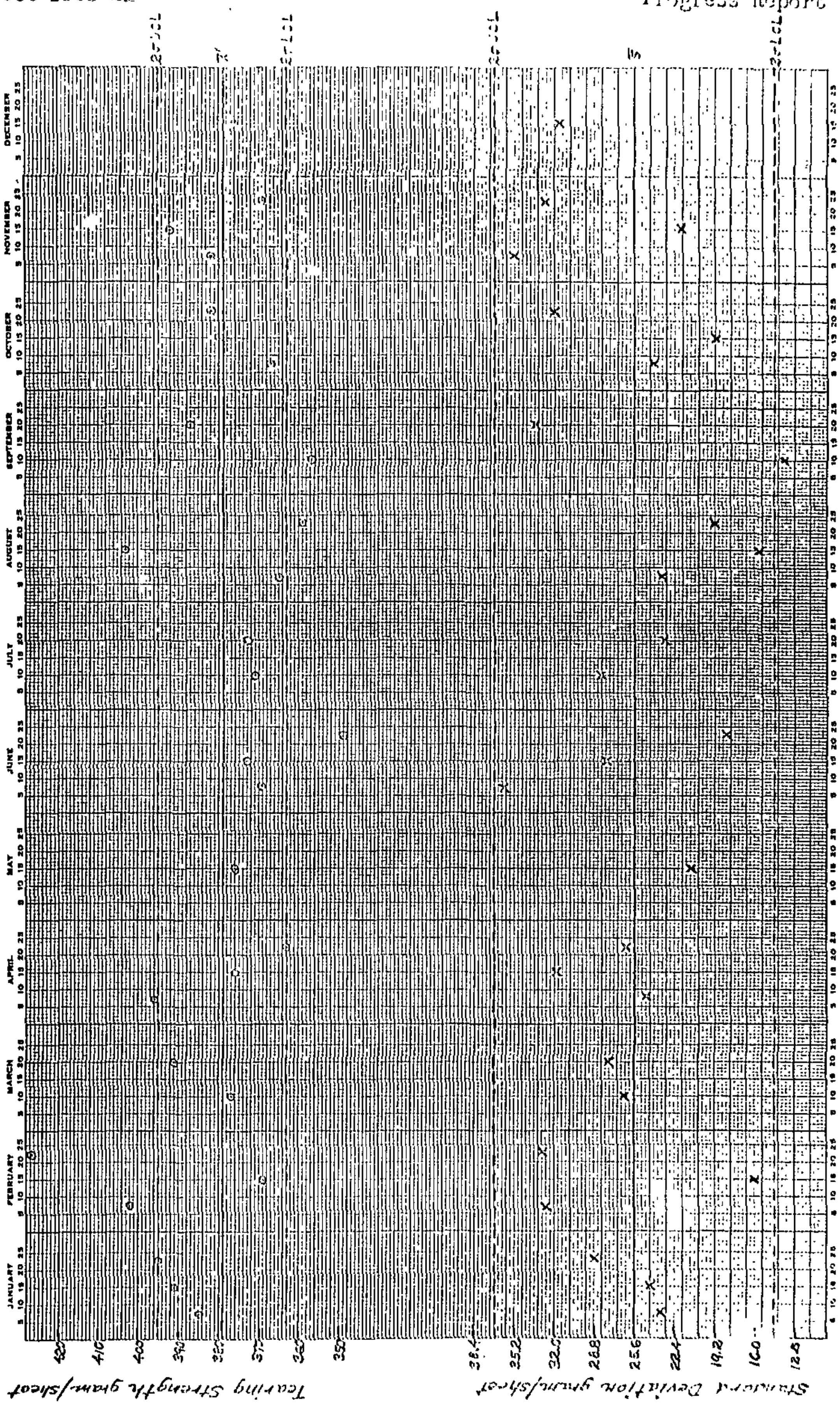

Figure 73

Cross-Machine Direction Elmendorf Tear--Mill F

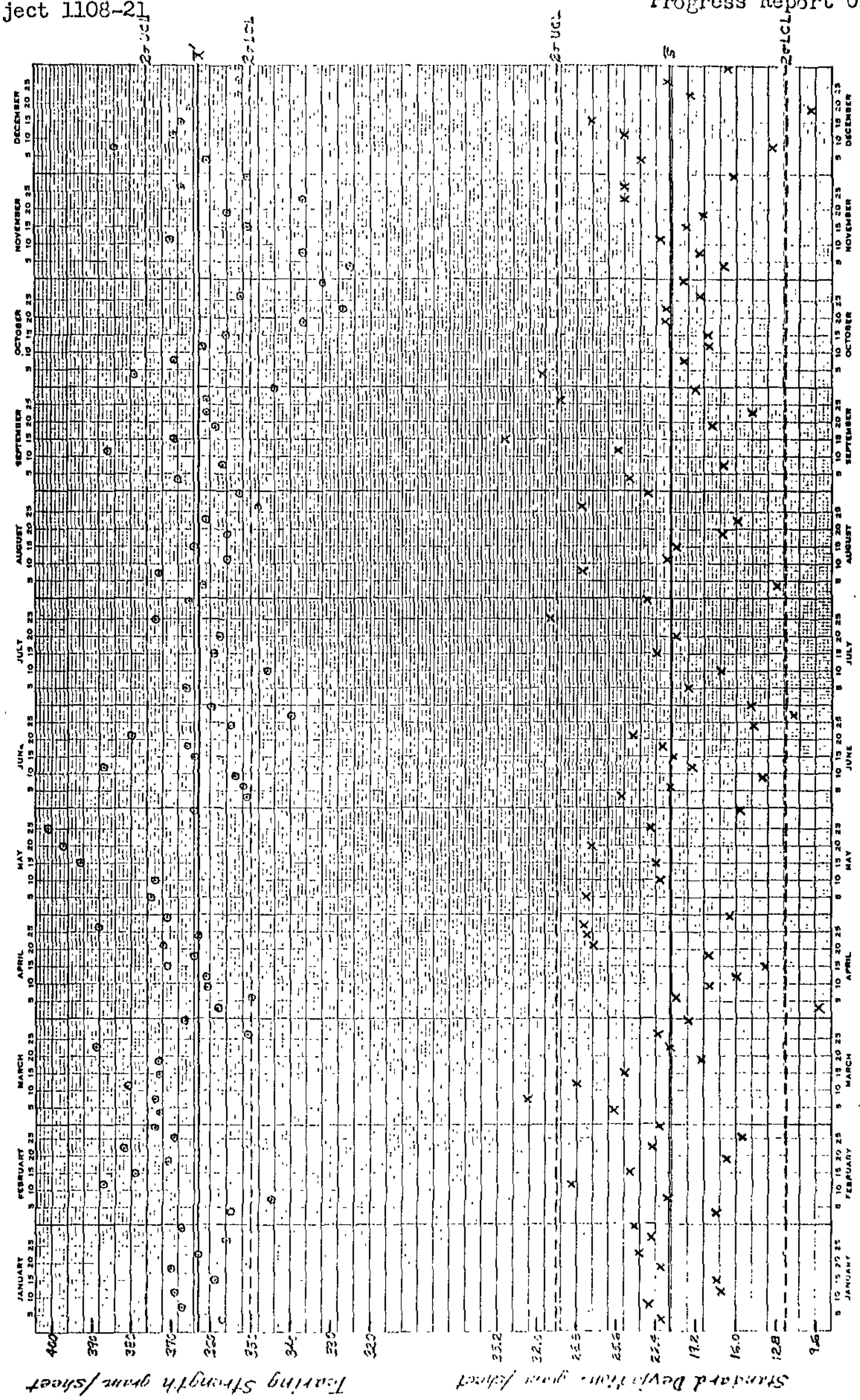

Figure 74

Cross-Machine Direction Elmendorf Tear--Mill G

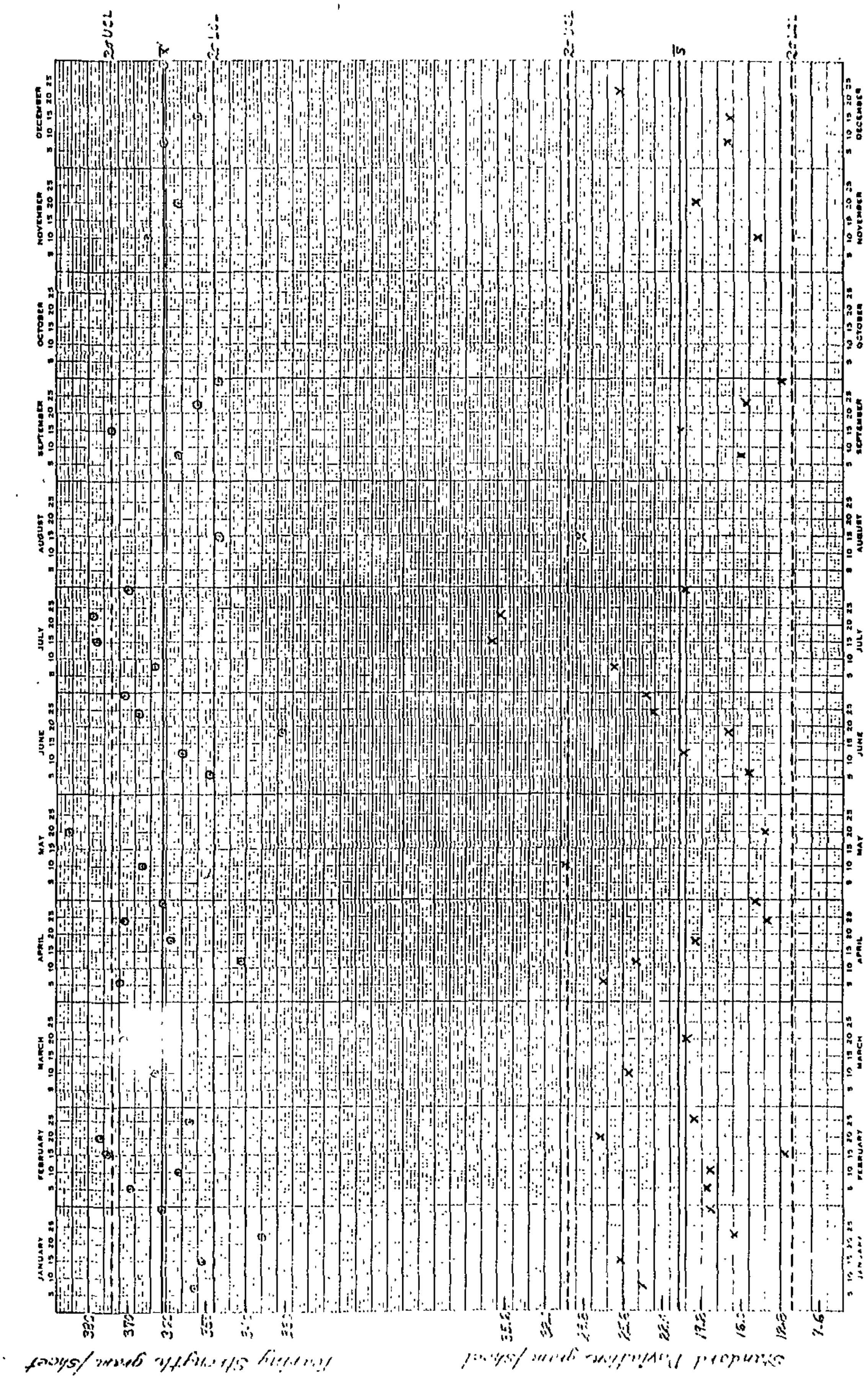

Figure 75

Cross-Machine Direction Elmendorf Tear--Mill II

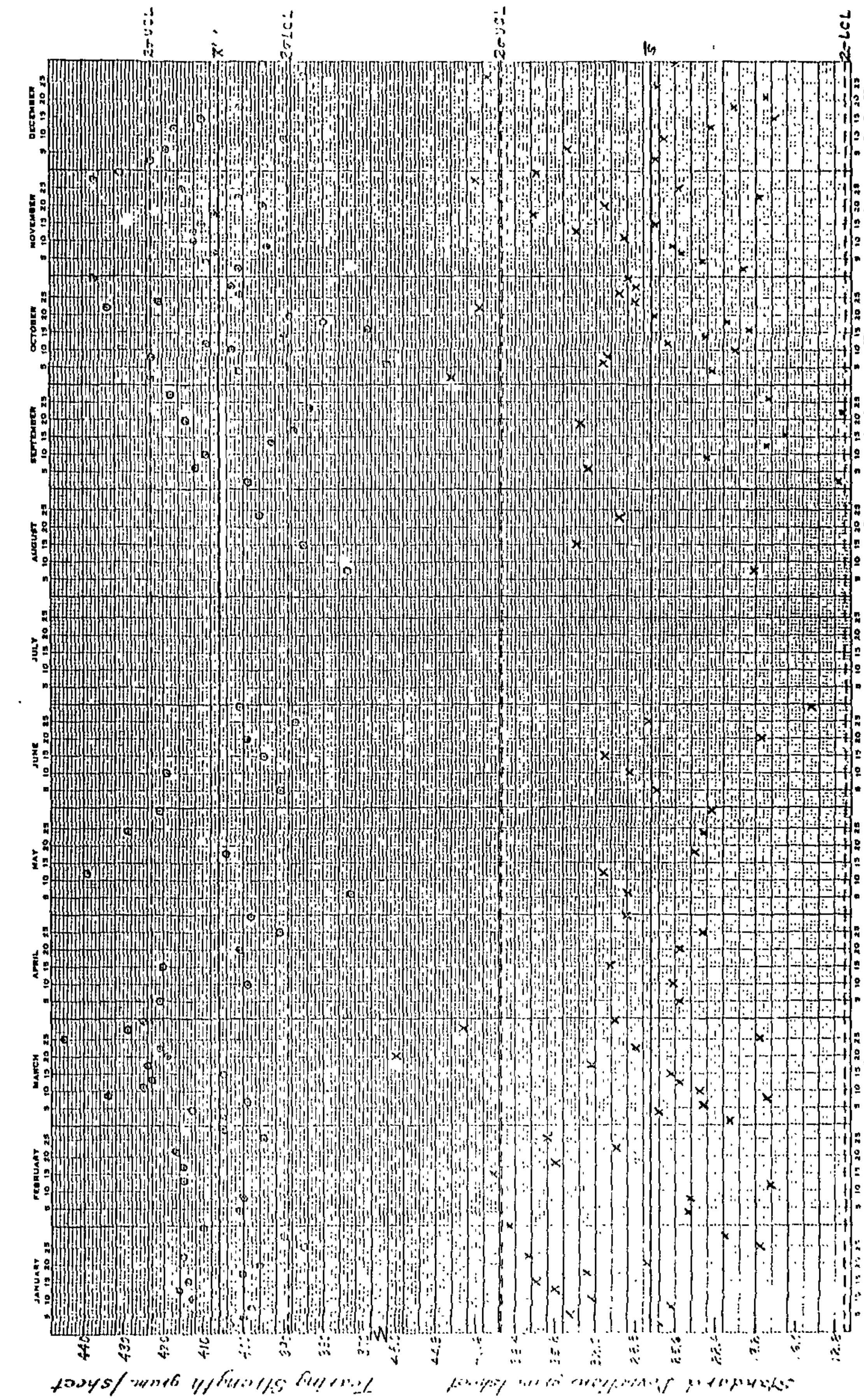

Figure 76

Cross-Machine Direction Elmendorf Tear--Mill I

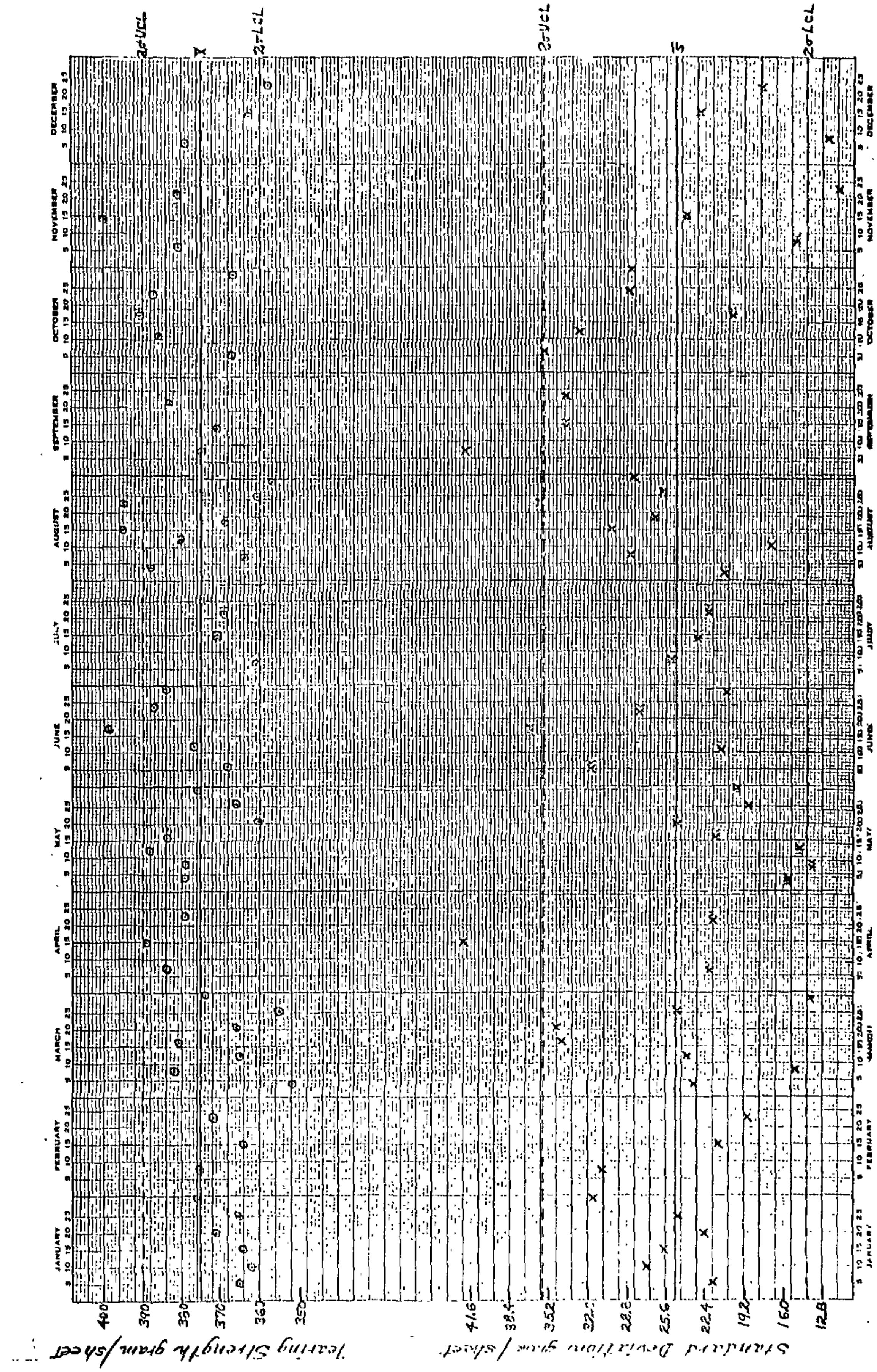

Figure 77

Cross-Machine Direction Elmendorf Tear—Mill J

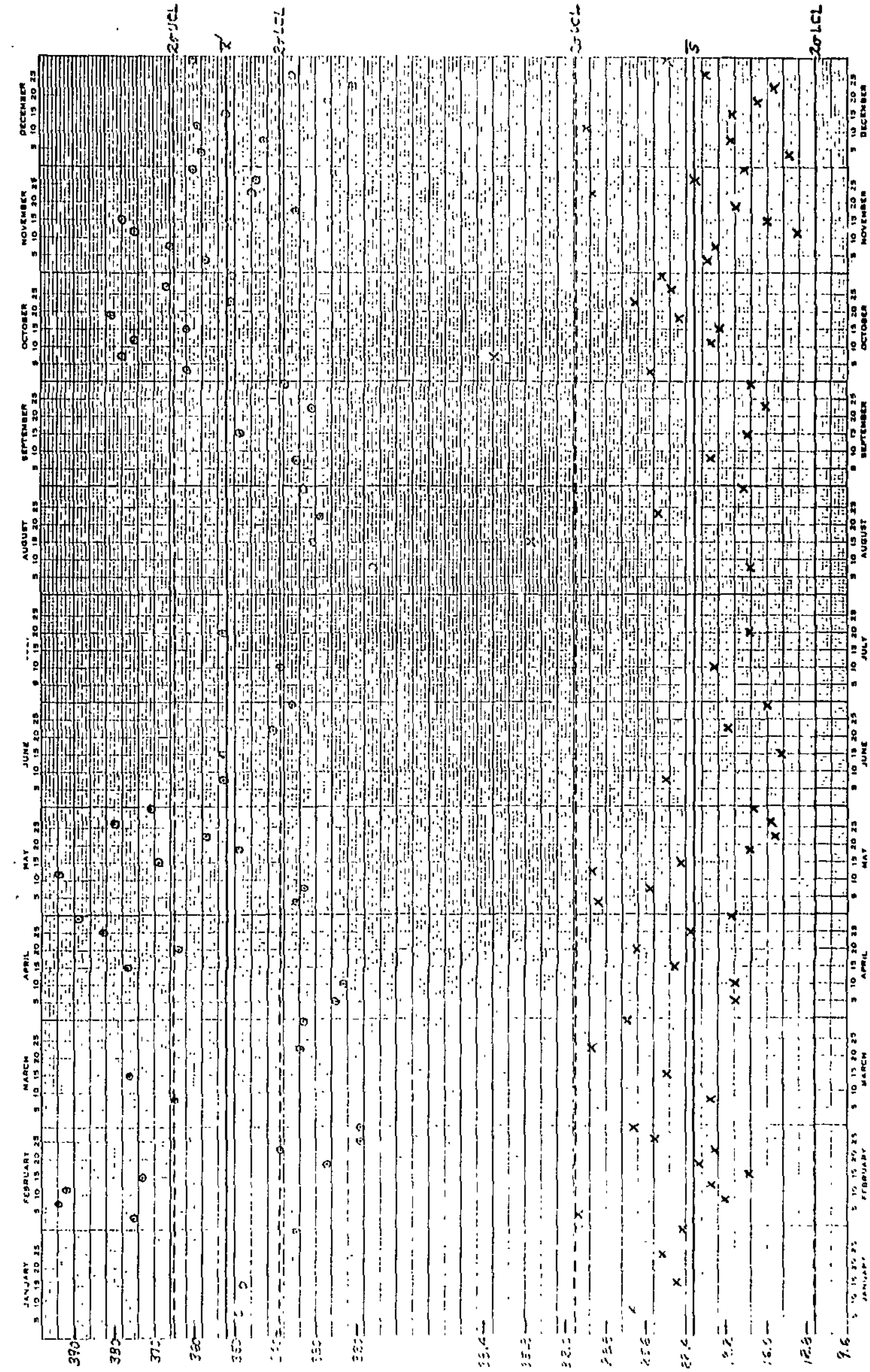

Figure 78

Cross-Machine Direction Elmendorf Tear--Mill K

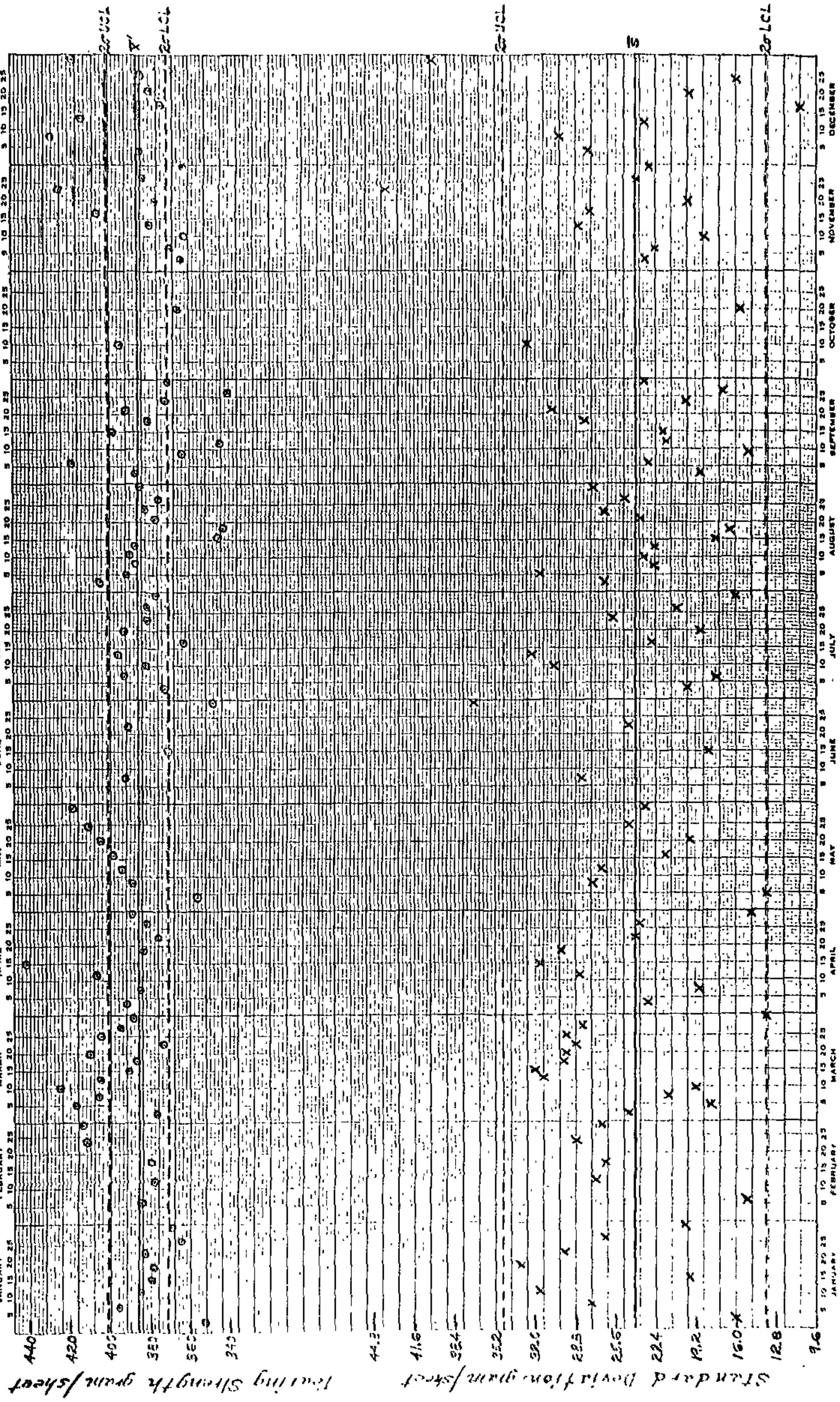

Figure 79

Cross-Machine Direction Elmendorf Tear—Mill L

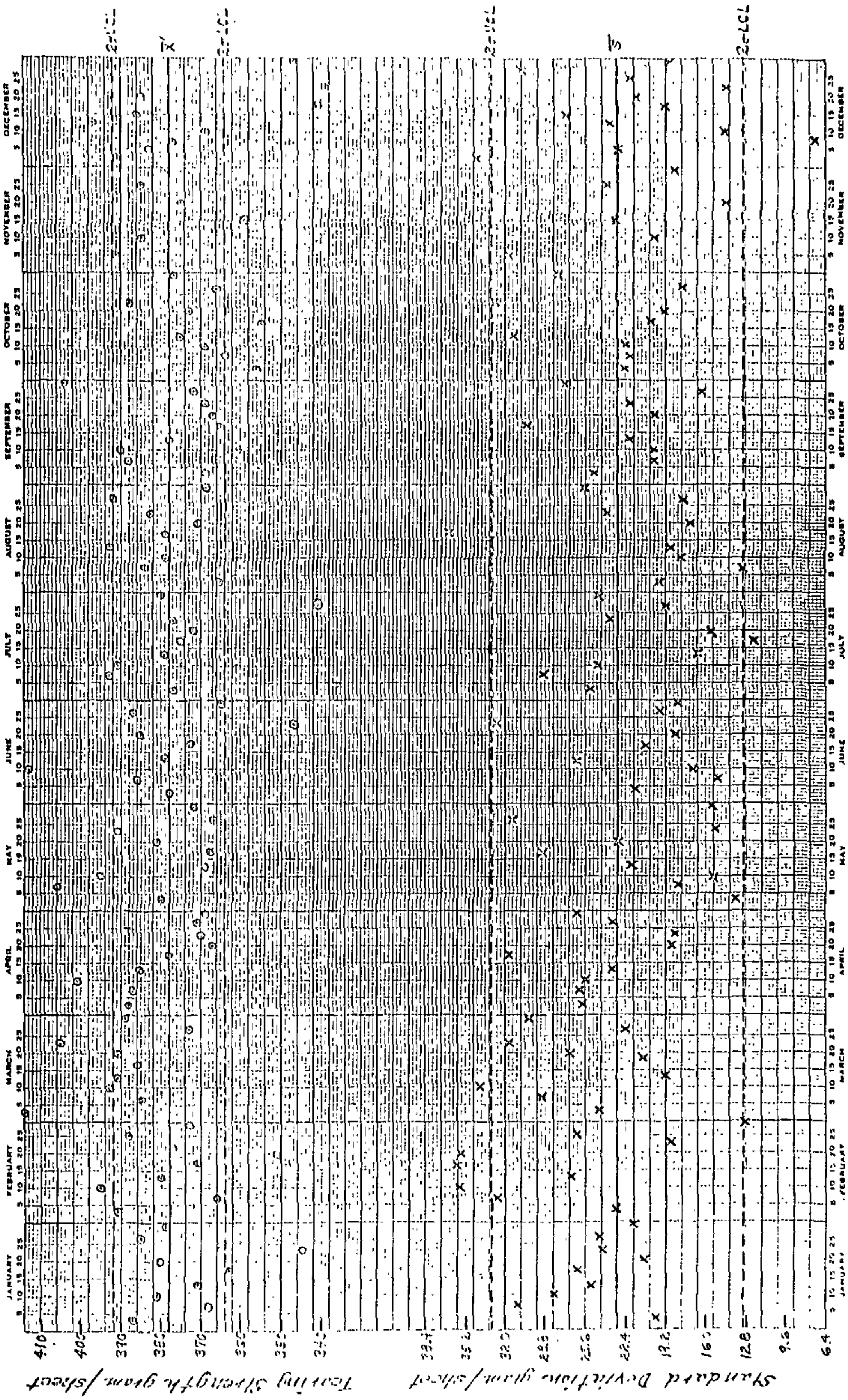

Figure 80

Cross-Machine Direction Elmendorf Tear---Mill M

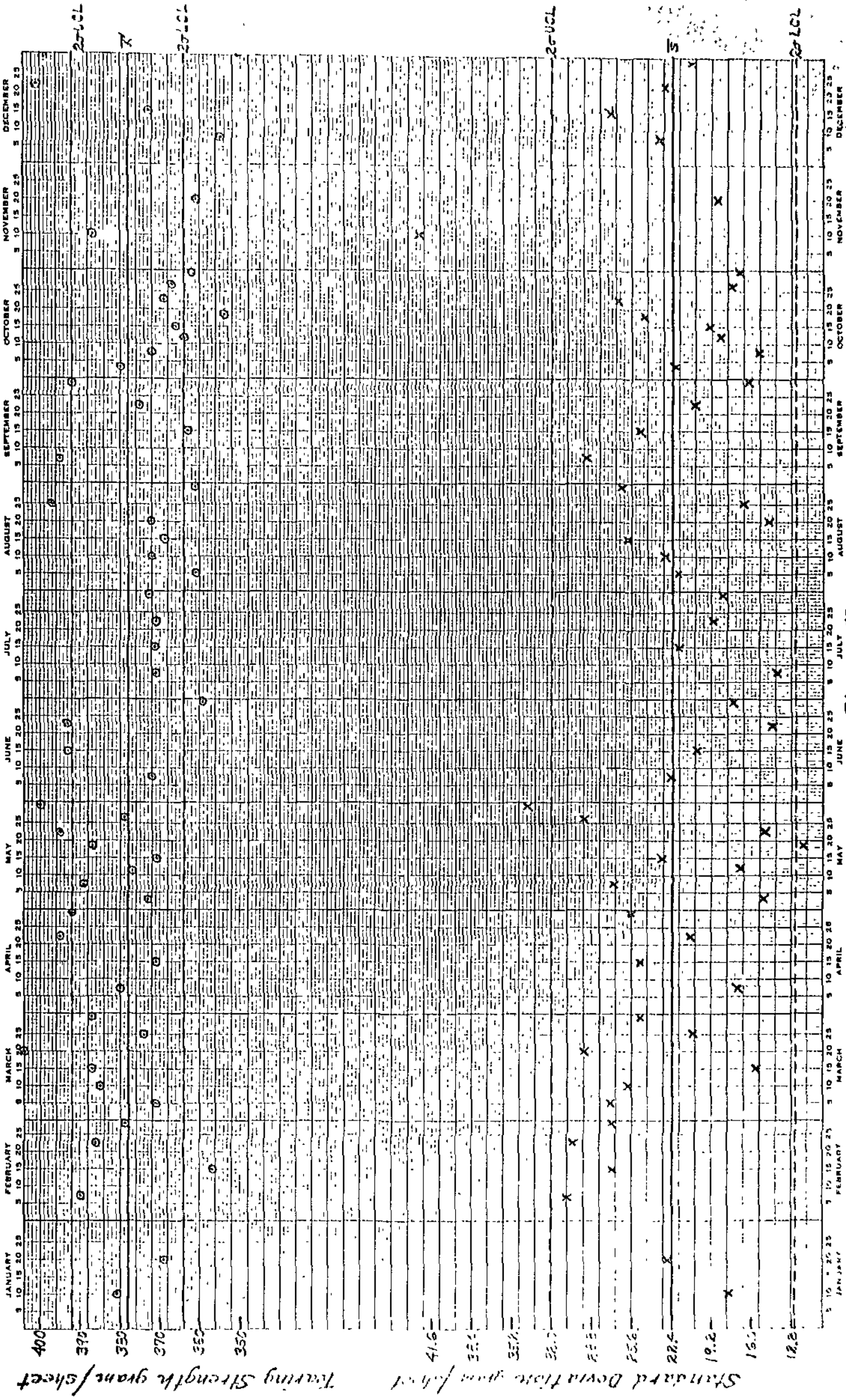

Figure 81

Cross-Machine Direction Elmendorf Tear--Mill N

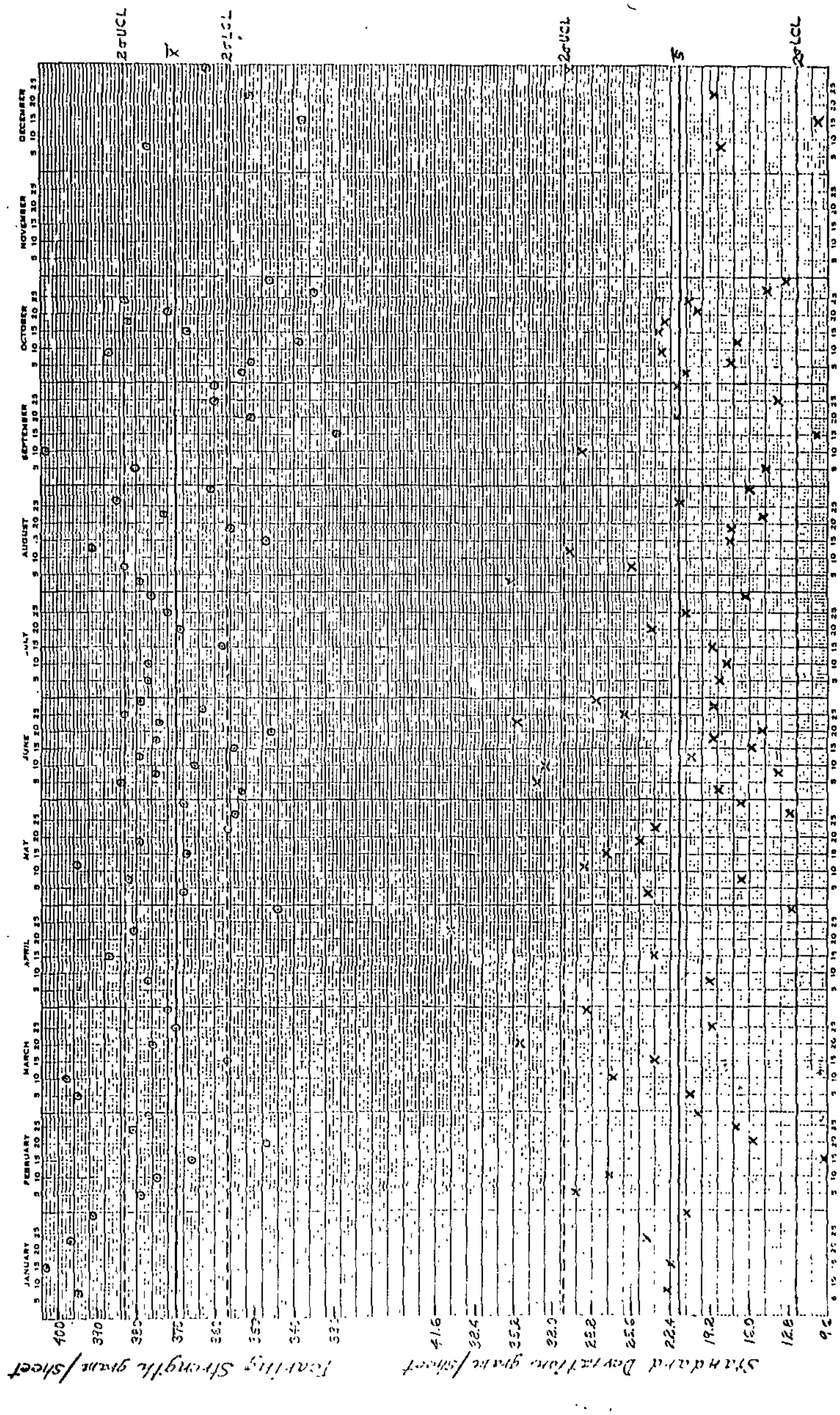

Figure 82

Cross-Machine Direction Elmendorf Tear--Mill O

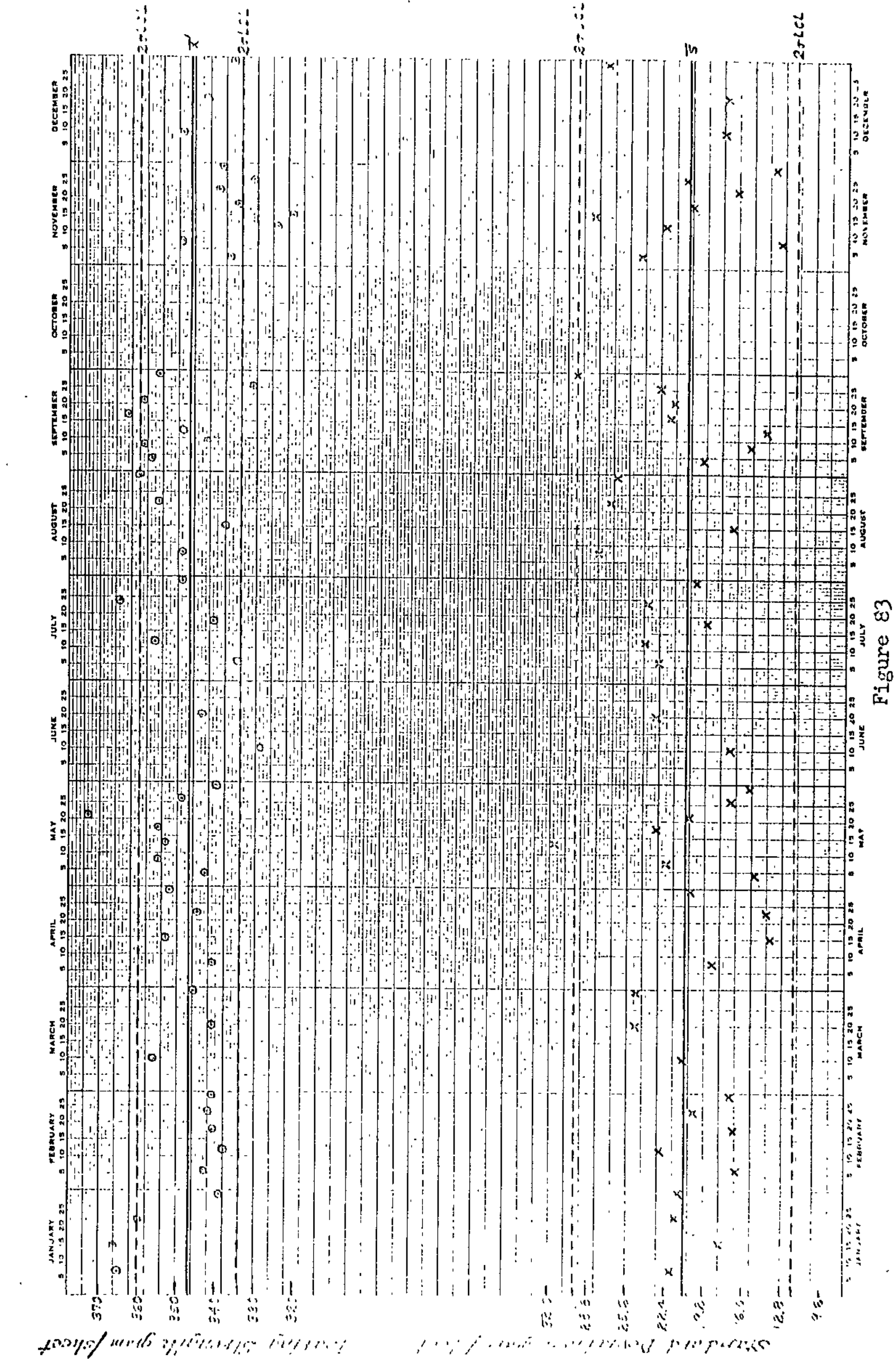

Figure 83

Cross-Machine Direction Elmendorf Tear--Mill P

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2. Duncan, A. J. Quality control and industrial statistics. Homewood, Ill., Richard D. Irwin Co., 1952.

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